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Article history: Received: 12.11.2022; Accepted: 25.02.2023; Published online: 25.03.2023

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Post-Brexit exchange rate volatility and its impact on UK exports to eurozone countries: A bounds testing approach

JEL Classification: C32; C58; E44; F14; F31; F41

Keywords: exchange rate volatility; exports; Brexit; ARDL; GARCH

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Abstract

Research background: The Brexit referendum had a profound effect on the economic relations between the United Kingdom (UK) and continental Europe. Major economic and financial determinants were affected, including the impact of the GBP/EUR exchange rate volatility on the dynamics of UK exports to the Eurozone.

Purpose of the article: This paper seeks to assess the extent to which these dynamics have changed since Brexit and to estimate the magnitude of their impact.

Methods: To this end, the volatility behavior of the GBP/EUR exchange rate before and after Brexit is captured using EWMA, GARCH(p,q), and EGARCH(p,q) models for the period of January 1, 2010 to August 31, 2020. The post-Brexit change in the volatility structure of GBP/EUR exchange rates is then tested by including a dummy in the optimal volatility model. Finally, the Autoregressive Distributed Lag (ARDL) Bounds Testing approach is employed to analyze the relationships between exchange rate volatility and exports.

Findings & value added: GARCH(1,1) was selected as the winning model and used to examine the volatility structure of the post-Brexit exchange rate, which revealed no significant change. By incorporating a well-grounded proxy for exchange rate volatility into the demand function of exports, and controlling for the industrial production index, terms of trade, and real exchange rate, the analysis showed that exchange rate volatility had a negative impact on export volume in both the long and short run. Additionally, the industrial production index had a positive effect on export volume in both the long and short run, while an appreciation in the value of the pound relative to the euro adversely affected the competitiveness of UK exports in the Eurozone market in the long run, with no impact in the short run. This paper serves as a benchmark for future studies, as it follows a three-step modeling approach and provides valuable insights into the potential economic and financial consequences a European Union (EU) member state may face should it choose to exit the EU.

Introduction

The European Union (EU) and the United Kingdom (UK) have enjoyed a rocky relationship since the 1960s. Continental Europe used all the means at its disposal to keep the UK in the bloc; it allowed the UK to bypass fiscal rules and the Schengen border, and reduced the British contribution to the common budget. The EU also eased the free movement of labor in the UK and allowed London to be the financial center of the Eurozone (Wang et al., 2017). The UK subsequently benefited not only from the free movement of goods and services with EU countries, but also from more than 38 EU trade agreements with non-European countries. The British Exit (Brexit) referendum vote clearly induced shock waves and uncertainty in the stock and exchange rate markets. Likewise, the actual exit in March 2019 added further turmoil to the UK’s exit plan and induced more uncertainty in the market, causing the GBP/EUR exchange rate to become more volatile. This increase in uncertainty is mainly attributed to the ambiguity surrounding
the UK’s future relationship with the EU, given the fact that the UK is one of the EU’s largest trading partners. Consequently, gauging the post-Brexit GBP/EUR exchange rate volatility structure and studying its impact on UK exports to Eurozone countries — specifically Austria, Belgium, Cyprus, Estonia, Finland, France, Germany, Greece, Ireland, Italy, Latvia, Lithuania, Luxemburg, Malta, Netherlands, Portugal, Slovakia, Slovenia, and Spain — is the ultimate purpose of this paper and addresses the scarcity of studies examining this particular feature related to Brexit.

Numerous theoretical and empirical studies have looked into how exchange rate volatility affects trade flows. In theoretical terms, there is no consensus on the impact of exchange rate uncertainty on exports as the relationship depends on traders’ appetite for risk and the existence of a developed forward exchange rate market (Broll & Eckwert, 1999; Dellas & Zilberfarb, 1993; Sercu & Vanhulle, 1992; Viaene & de Vries, 1992).

The majority of empirical studies have found that trade flows are negatively impacted by exchange rate volatility due to adjustment costs and uncertainty for exporting investors who are risk-averse, supporting the trade theory (Arize, 1995; Bahmani-Oskooee & Gelan, 2018; Hayakawa & Kimura, 2009). Few studies, however, have found a positive link between exchange rate volatility and trade flows (Asseery & Peel, 1991), while others found no statistically significant impact on trade volume (Aristotelous et al., 2001; Nishimura & Hirayama, 2013). These inconsistent results are a consequence of the characteristics of the industry (Bahmani-Oskooee & Aftab, 2017), the income of the import partner (Chi & Cheng, 2016), and the statistical techniques used to estimate exchange rate volatility.

In light of the previous studies, this paper attempts to give an answer to a question currently of crucial interest in the framework of international economics, as it relates to the effects of Brexit on the GBP/EUR exchange rate volatility and its impact on UK exports. Specifically, our methodological approach consists of applying a three-step strategy to a demand function of exports whose drivers are relative prices, income, and volatility, in line with the consensus in the literature.

First, in order to measure and model the volatility behavior of the GBP/EUR exchange rate, the Exponentially Weighted Moving Average (EWMA), the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) (1,1) and the Exponential Generalized Autoregressive Conditional Heteroskedastic (EGARCH) (1,1) models are implemented for the period from January 1, 2010 to August 31, 2020. The best model is then
used to check whether the volatility structure of GBP/EUR exchange rates has changed following the Brexit referendum (June 24, 2016 — August 31, 2020). Then, the volatility (VOL) dynamics provided by the selected model are added to an export demand function that includes the industrial production index of the export destination countries (IPI), the real effective exchange rate (REER), and the terms of trade (TOT), in order to examine their impacts on exports for the period extending from January 2010 till August 2020. For this purpose, and given the uncertainty related to the order of integration of the volatility measure, we follow an ARDL bounds testing cointegration approach.

The paper is organized as follows: In Section 2, we present a comprehensive literature review. Section 3 outlines the econometric models and methodology employed, and describes the data. Section 4 presents the results, Section 5 delves into the implications of the findings, and Section 6 concludes with a summary of the key takeaways.

**Literature review**

Theoretically and empirically, it is unclear how exchange rate volatility and exports are related. Clark (1973) argued that exchange rate fluctuation has a detrimental effect on exports. According to his proposed model, the market is perfectly competitive with one single traded good for a given export market, investors are risk averse, hedging possibilities are extremely limited, and adjustment costs are present. Risk-averse firms would reduce their risk exposure, which would result in a decrease in exports. They would do so in an effort to enhance their expected utility, which depends on both the value and variance of earnings. Some of these assumptions were relaxed in later studies (Cushman, 1986; Viaene & de Vries, 1992), which led to a reduction in exposure to risk; however, the possibility of the same negative impact of exchange rate volatility on exports still prevails. By the same token and in conditions of high volatility, risk-averse merchants may trade less because they run the risk of incurring unforeseen expenses related to changes in exchange rates (Doğanlar, 2002). However, for firms that can hedge their contracts using a well-developed forward exchange rate market, higher exchange rate volatility would not hamper exports. Grauwe (1988) argued that the assumption of risk-averse investors is not sufficient to induce a negative relationship between exchange rate
volatility and exports. According to the author, what matters is the degree of risk aversion since an increase in risk has substitution and income offsetting effects. The substitution effect lowers the expected utility, which discourages risk-averse firms from exporting, while the income effect encourages agents to increase their exports to avoid the possibility of a severe decline in revenues. Thus, the impact of volatility on exports depends on which effect dominates (Goldstein & Khan, 1985).

Aristotelous et al. (2001) examined the effects of various exchange rate volatilities and regimes (fixed, floating, and managed-float regimes) on British exports to the United States from 1889 to 1999. Using a generalized gravity model, the authors found that exchange rate volatility and exchange rate regimes had no impact on these exports. Three approaches were employed by Umaru et al. (2013) to investigate the impact of the volatility in exchange rate on Nigerian exports, including Ordinary Least Squares (OLS), Granger-Causality, and Generalized Autoregressive Conditional Heteroskedasticity (GARCH). They found evidence of a negative impact of exchange rate volatility on exports. Serenis and Tsounis (2013) reported the same negative relationship in the context of Cyprus and Croatia, and the same adverse effect on exports was confirmed in other studies covering Asian countries (Aftab et al., 2017; Baek, 2014; Nishimura & Hirayama, 2013; Pino et al., 2016; Safuan, 2017). Aftab and Rehman (2017) used industry-level disaggregated data for Malaysia and Singapore and an Autoregressive Distributed Lag (ARDL) model to investigate the influence of exchange rate volatility on international trade in the period 2000–2014. Their results indicate that the influence of exchange rate risk varies between industries for different periods. Chi and Cheng (2016) confirmed the significant impact of exchange rate volatility, which was measured using mean adjusted relative change, on the export volume of the maritime sector in Australia to its trading partners in Asia.

According to Bahmani-Oskooee and Aftab (2017), the majority of Malaysian industries’ trade flows are significantly impacted by exchange rate volatility, albeit with an asymmetrical effect. While some industries benefit from increased volatility, others benefit from a reduction in volatility. Sharma and Pal (2018) estimated exchange rate volatility using autoregressive conditional heteroskedasticity models and determined the short- and long-run relationships using pooled mean group estimators. They found that volatility has a dampening effect on India’s exports to the United States, Germany, and China, and on imports from the United States and
China. Smallwood (2019) analyzed the impact of exchange rate volatility on bilateral Chinese exports to 10 markets. Using the DCC-GARCH model to estimate exchange rate volatility, he found that export growth is negatively related to volatility for the majority of countries, except the US market. Volatility had no significant impact on Chinese exports to the United States. Bahmani-Oskooee and Kanitpong (2019) estimated linear and non-linear ARDL and found a short-term asymmetric effect on the trade flows between Thailand and China. Sugiharti et al. (2020) estimated volatility using the GARCH model and found that exchange rate volatility significantly affects some Indonesian exports of commodities to South Korea, Japan, India, and the United States. The impact of exchange rate volatility was also found in the short and long run in a US-China context (Hurley & Papanikolaou, 2021). Chi (2020) found that exchange rate volatility has an asymmetric effect on the freight flows between the United States and Canada.

Recently, Bahmani-Oskooee and Saha (2021) found that currency volatility has a partner-specific impact on India’s imports and exports. Conversely, Choudhry (2008) investigated the role of exchange rate volatility in the imports to the UK from Canada, Japan, and New Zealand for the period spanning 1980 to 2003, using a Johansen multivariate cointegration method and a constraint error correction model. They concluded that real imports are significantly positively affected by the exchange rate fluctuations. Similarly, a positive impact was also observed for the cases of Germany-US bilateral trade flows (McKenzie & Brooks, 1997) and Australia’s exports to its main Asian partners (Chi & Cheng, 2016). Aftab et al. (2016) used a bounds testing approach to cointegration and found that the exchange rate risk encouraged trade flows between Malaysia and Japan.

However, according to Bahmani-Oskooee et al. (2016), there is no significant effect of exchange rate changes on Pakistani and Japanese trade. Similarly, Asteriou et al. (2016) utilized GARCH, ARDL, and Granger causality to investigate the effect of a volatile exchange rate on international trade for Mexico, Indonesia, Nigeria, and Turkey. They concluded that, except for Turkey, the exchange rate has no significant long-run impact on international trade.

Bahmani-Oskooee et al. (2018) investigated the impact of exchange rate uncertainty on the trade between the UK and the United States at the commodity level. Using 67 industries, they found that 18 British industries exporting to the United States saw short-term effects, but just 15 businesses
experienced long-term consequences. When a non-linear model was estimated, the short-run asymmetric effect was found for 43 industries, which lasted into the long run in 33 industries. Similarly, using trade flows between the UK and Germany, Bahmani-Oskooee and Karamelikli (2022b) found that 36 exporting industries experienced a strong short-term impact of exchange rate fluctuations, which persisted into the long term in 23 industries. They also found short-run asymmetric effects of volatility. Finally, Bahmani-Oskooee and Karamelikli (2022a) assessed the sensitivity of bilateral trade between the UK and China to exchange rate fluctuations. They found that as exchange rate volatility increased, it discouraged UK exports to China in most industries and encouraged Chinese exports to the UK in most industries.

Research method

As outlined in the introduction, our approach to estimate the impact of the GBP/EUR exchange rate volatility on UK exports to Eurozone countries following the Brexit vote consists of three steps.

Step 1: Modeling volatility using EWMA and GARCH Models

The literature on exchange rate volatility reveals that three main measures of volatility are used: the standard deviation and its moving average (Arize et al., 2000; Chowdhury, 1993; Nishimura & Hirayama, 2013), and GARCH specifications (Bahmani-Oskooee & Aftab, 2017; Sharma & Pal, 2018). Since heteroskedasticity, volatility clustering, and leptokurtosis are considered stylized facts of the exchange rate time series. We opted to use GARCH(1,1) and EGARCH(1,1) models given their ability to capture a wide range of volatility dynamics, including mean-reverting or explosive behavior, as well as their capacity to model time-varying volatility and asymmetric volatility, which is often observed in exchange rate data. This is further supported by empirical evidence, as many studies have found that GARCH (1,1) and EGARCH (1,1) models provide improved estimates of exchange rate volatility compared to other models (Naimy et al., 2020). We also chose to use EWMA, given its efficiency, simplicity (Naimy & Hayek, 2018) and performance in predicting fiat currencies’ volatility. Model selection is based on the three error-based metrics; namely, Root Mean Square Error.
(RMSE), Mean Absolute Error (MAE), and Mean Absolute Percentage Error (MAPE).

EWMA

The EWMA model is based on the conventional concept of variance, but involves the realistic assumption that older observations should be given less weight than more recent ones. More specifically the weights decrease exponentially as we move away from the prediction period. Accordingly, assuming that we use \( n \) observations to estimate the non-constant variance, and that returns are assumed to be zero-mean:

\[
\sigma_t^2 = \sum_{i=t-1}^{t-n} \alpha_i x_i^2, \quad \sum_{i=1}^{n} \alpha_i = 1 \quad i > j \Rightarrow \alpha_i > \alpha_j
\]

with

\[
\alpha_i = (1 - \lambda)\lambda^{i-1} = \lambda \alpha_{i-1}, \quad 0 < \lambda < 1,
\]

so that

\[
\sigma_t^2 = \sum_{i=t-1}^{t-n} (1 - \lambda)\lambda^{(t-1)-i}x_i^2 = \sum_{i=1}^{n} \alpha_i x_i^2 \quad \text{with} \quad \alpha_i = (1 - \lambda)\lambda^{(t-1)-i}
\]

where \( \sigma_t^2 \) is the variance of today, \( \sigma_{t-1}^2 \) is the variance of the previous day and \( \lambda \) is the smoothing parameter and decay factor, ranging between 0 and 1.

After some algebraic manipulations, the above equation transforms into the well-known expression

\[
\sigma_t^2 = \lambda \sigma_{t-1}^2 + (1 - \lambda)x_{t-1}^2,
\]

where \( x_{t-1}^2 \) is the square of the previous day’s return.

GARCH(1,1)

When including a long-run variance in the EWMA model, the GARCH(1,1) model is obtained, which is a narrow form of the more gen-
eral GARCH\((p,q)\) model. GARCH(1,1) and GARCH\((p,q)\) are defined as in Equations (2) and (3), respectively:

\[
\sigma_t^2 = \left( \frac{\omega}{1 - \alpha - \beta} \right) V_L + \alpha x_{t-1}^2 + \beta \sigma_{t-1}^2, \quad (2)
\]

\[
\sigma_t^2 = \left( \frac{\omega}{1 - \alpha - \beta} \right) V_L + \sum_{i=1}^{p} \alpha_i x_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2, \quad (3)
\]

where \(\sigma_{t-i}^2\) and \(x_{t-i}^2\) represent the variance and the squared return the \(i\)-th day before today, respectively, and \(V_L\) is the long-run variance rate. \(\gamma, \beta, \) and \(\alpha\) are the model coefficients. The model is deemed stable when these three weights equal one. The constraint of \(\alpha + \beta < 1\) guarantees the covariance stationarity. \(\alpha + \beta\) represents the persistence of the conditional volatility as their sum determines the pace of the conditional variance slowly moving back to the long-run variance after it deviates from its original value. The persistence of the shocks is determined by \(\beta\).

It is worth noting that the weights of the past returns in GARCH (1,1) decay exponentially.

\[
\sigma_t^2 = \omega + \alpha x_{t-1}^2 + \beta \sigma_{t-1}^2
\]

\[
= \omega + \alpha x_{t-1}^2 + \beta \left( \omega + \alpha x_{t-2}^2 + \beta \sigma_{t-2}^2 \right)
\]

\[
= \omega + \beta \omega + \alpha x_{t-2}^2 + \alpha \beta x_{t-2}^2 + \beta^2 \sigma_{t-2}^2
\]

\[
= \omega + \beta \omega + \alpha x_{t-2}^2 + \alpha \beta x_{t-2}^2 + \beta^2 \left( \omega + \alpha x_{t-3}^2 + \beta \sigma_{t-3}^2 \right)
\]

\[
= \omega + \beta \omega + \beta^2 \omega + \alpha x_{t-2}^2 + \alpha \beta x_{t-2}^2 + \alpha \beta^2 x_{t-3}^2 + \beta^3 \sigma_{t-3}^2
\]

\[
............
\]

where \(\omega = \gamma V_L\).
EGARCH(1,1)

Although GARCH(1,1) accounts for relevant stylized facts of financial series including volatility clustering, skewness and the presence of extreme values, it fails to capture the leverage effects, which is addressed by the second generation GARCH models such as the EGARCH(1,1). The EGARCH(1,1) (Nelson, 1991) is asymmetric by nature and accounts for the leverage effect in the sense that the conditional variance is influenced by both the sign and the size of the lagged innovations. In this way, the model explains how volatility reacts differently to the same-sized upwards and downwards movements of the series. The natural logarithm of the variance is modeled by the EGARCH specification, rather than the variance itself. As a result, sign restrictions on the model parameters are not required to ensure that the conditional variance is positive.

More specifically, EGARCH implements a function $g(e_t)$ of the innovations, $e_t$, where the innovation values are determined by $|e_t| - E|e_t|$.

$$
\log \sigma_t^2 = \frac{\sum}{1} - \alpha - \beta \right) V_t + \theta g(e_{t-1}) + \beta \log \sigma_{t-1}^2, \\
\text{with } g(e_{t-1}) = \delta e_{t-1} + \alpha(|e_{t-1}| - E|e_{t-1}|). \tag{5}
$$

In the case of Gaussianity, EGARCH (1, 1) is:

$$
\log \sigma_t^2 = \omega + \beta \log \sigma_{t-1}^2 + \delta e_{t-1} + \alpha \left( |e_{t-1}| - \frac{2}{\sqrt{\pi}} \right). \tag{6}
$$

**Step 2: Testing for a change in the post-Brexit volatility structure of GBP/EUR exchange rates**

To test whether the Brexit referendum has changed the volatility structure of the GBP/EUR exchange rates, models (1), (2) and (5) are extended by adding a dummy (Dt) that takes the values 0 (for the pre-Brexit referendum period, January 1, 2010 to June 23, 2016) and 1 (for post-Brexit referendum, June 24, 2016 to August 31, 2020). The extended models (1), (2) and (5) are presented in (7), (8) and (9).

---

1 Innovations are the standardized errors divided by the conditional standard deviations.
The dummy variable is intended to provide information on the change in the volatility of the GBP/EUR exchange rate through its sign, and mainly through its significance. More specifically, in the case of a significant value of $\phi$, a positive (negative) sign of this parameter signals that the volatility has increased (decreased) after the Brexit referendum.

**Step 3: Modeling the Impact of GBP/EUR Exchange Rate Volatility on UK Exports Using the ARDL Approach**

The volatility series (VOL), having been determined by the volatility model that best captures the volatility dynamics of the GBP/EUR exchange rates, is added to the conventional set of drivers in the traditional export demand function. As described in the introductory section, these drivers are IPI, REER, and TOT, (Ekanayake et al., 2010; Matsubayashi & Hamori, 2003; Safuan, 2017).

It is worth noting that IPI is used as a proxy for GDP, since IPI is available on a monthly basis, while GDP is available on a quarterly basis. The IPI of the export destination countries (the Eurozone countries) has been computed as the weighted average industrial production index of these countries (the weights are based on the volume of exports to the country in question). TOT refers to a price index measured as export prices index divided by import prices index. Thus, it is the amount of imports that can be exchanged for one unit of export. An improvement in TOT may ameliorate the standard of living in a country. When TOT values are more favorable, the incentive to invest in the export sector is greater. Since the export price index and import price index of the UK provided by the International Monetary Fund (IMF) database are only available until June 2019, we have used the Commodity Terms of Trade as a proxy for TOT, which was calculated until September 2020. In fact, it is a country-specific commodity price index, including the price of 45 individual commodities, weighted using commodity-level trade data. More specifically, the weight of each commodity is the share of net exports in output ($ exports of commodity i minus $
imports of commodity \( i \)/(GDP). The impact of Commodity Terms of Trade on macroeconomic outcomes, including GDP and consumption, has been investigated. Its effect will likely depend on whether the increase in the index is because of an increase in the price of the commodity the country exports or a decline in the price of a commodity it imports. The former case would lead to an increase in investment as a result of increased profitability. Aggregate consumption would also tend to increase (Gruss & Kebhaj, 2019). Finally, REER is defined as the nominal exchange rate deflated by the corresponding price indices. Therefore, in our case, REER compares the real value of the pound against the euro.\(^2\) An increasing REER indicates that a country is losing its competitive edge.

Equation (10) shows the extended traditional demand function for exports, whose data generating process is given by:

\[
EX_t = a_0 \cdot IPI_t^{\lambda_1} \cdot REER_t^{\lambda_2} \cdot TOT_t^{\lambda_3} \cdot VOL_t^{\lambda_4} \cdot \epsilon_t. \tag{10}
\]

Log-log transformation is used for estimation purposes:

\[
LEX_t = \log a_0 + \lambda_1 \cdot L\text{IPI}_t + \lambda_2 \cdot L\text{REER}_t + \lambda_3 \cdot L\text{TOT}_t + \lambda_4 \cdot L\text{VOL}_t + \epsilon_t. \tag{11}
\]

Where “\( L \)” means log, \( EX \) denotes the volume of UK exports to the Eurozone countries and \( \epsilon_t = \log \epsilon_t \) is an error term.

The signs of the lambda coefficients are expected to be as follows. According to the gravity theory of international trade, an increase in the IPI of trading partners is predicted to boost the volume of exports; therefore \( \lambda_1 \) is expected to be positive and significant. On the other hand, according to the relative price effect, REER may result in a rise in the volume of exports, so \( \lambda_2 \) is expected to be positive and significant (Thuy & Thuy, 2019). Similarly, a higher value for TOT, indicating an increase in the price of exports as compared to the price of imports, might reduce the volume of exports, so it is expected to have a significant negative value for \( \lambda_3 \). Finally, there is no clear relationship between exchange rate volatility and export, so \( \lambda_4 \) could be significantly positive or negative, or even not significant. As for the value of the model coefficients, it must be interpreted as the expected percentage change in the UK exports to the Eurozone associated with a 1% in-

\(^2\) In general, the REER compares the value of a country’s currency against the weighted average of the currencies of its main partners. However, in our case, the currency of all the partners is the euro.
crease in the corresponding covariate. In economic terms, lambda coefficients are elasticities.

The ARDL cointegration approach developed by Pesaran et al. (2001), or bound cointegration technique, is implemented to determine the long-run relationship between the non-stationary series involved in the UK export demand function. Then, it is reparametrized to the Error Correction Model (ECM) to also estimate the short-run dynamics of the underlying variables. Accordingly, the short-run dynamics are integrated with the long-run equilibrium.

As is well known, this approach has been widely used mainly because of six important features: (i) Its advantages when considering cointegration relationships (Thuy & Thuy, 2019) regardless of whether the regressors involved are I(0) or I(1) or a combination of both. More specifically, the ARDL technique for cointegration produces accurate and reliable estimates. This is a remarkable advantage given the low power of the unit root tests for non-stationarity. (ii) Its efficiency in dealing with small and finite sample data. (iii) Endogeneity is not a problem due to the absence of residual correlation (i.e. all variables are assumed to be endogenous). Consequently, the bounds testing is performed on each variable as endogenous variables while others are presumed to be exogenous. (iv) It identifies the cointegrating vectors when there are several of them (this is a particularly important advantage). And finally, (v) It allows for error correction and for variables to be assigned different lag lengths without affecting the distribution of the test statistic. Through a straightforward linear transformation, the ECM model may be created from the ARDL model. It integrates short-run adjustments with long-run equilibrium while preserving long-run information. The associated ECM model has enough lags to accurately represent the general data generating process to specific modeling frameworks (Nkoro & Uko, 2016).

Data

The GBP/EUR daily exchange rates were extracted from the Thomson Reuters’s database for the period from January 1, 2010 to August 31, 2020, totaling 2,782 daily observations, and were used to find the volatility model that best captures the volatility dynamics of the GBP/EUR exchange rates. Monthly data for IPI, REER, TOT, and the UK’s exports to Eurozone coun-
tries were collected from the IMF database for the same period, totaling 128 observations. VOL data were the estimated time series provided by the GARCH-type model selected in step 1 using GBP/EUR monthly exchange rates.

Table 1 depicts the summary statistics for the exchange rate simple return in Panel A and for the regression variables in Panel B, and Figure 1 displays the time series plot of the daily GBP/EUR exchange rate returns, where volatility clustering is clearly illustrated.

Results

Selection of the Volatility Model for the GBP/EUR exchange rate dynamics

The parameter estimates resulting from the estimation of the EWMA, GARCH(1,1), and EGARCH(1,1) specifications are presented in Table 2. The Student’s t distribution was selected based on the maximum likelihood and lowest Akaike info criterion (AIC), Schwarz criterion (SCIC), and Hannan-Quinn (HQIC) criterion. The estimate for the smoothing parameter in the EWMA model (λ) suggests a stable volatility of the GBP/EUR exchange rate, and the ARCH effect (α) under the GARCH and EGARCH (7% and 15%, respectively), indicates a significant (p=0.0000) but moderate reaction of volatility to shocks. The GARCH effect (β) is quite notable, suggesting strong (but not explosive) volatility persistence. On the other hand, the negative sign of γ (p=0.0490) indicates a significant leverage effect in the variance generating process where a negative shock is more likely to affect volatility than a positive shock. Figure 2 plots the realized volatility against the three selected volatility models; GARCH(1,1) exhibited the highest volatility surrounding the time of Brexit referendum as compared to EWMA and EGARCH(1,1).

Table 3 reports the values of the key error-based performance indicators for EWMA, GARCH(1,1) and EGARCH(1,1) models, along with their rankings. According to RMSE and MAE, along with the value of the log likelihood function, the GARCH(1,1) specification is the one that best models the volatility dynamics of the GBP/EUR exchange rate.
Investigating whether the Brexit referendum can be considered an exogenous break point in the dynamics of the GBP/EUR exchange rates series

As can be seen in Table 4, $\phi$ is insignificant in the extended GARCH(1,1) specification, the one selected in subsection 3.1 to model the volatility of the GBP/EUR exchange rate during the period under analysis. This finding indicates that the Brexit referendum cannot be considered an exogenous break point in the dynamics of the GBP/EUR exchange rate and, accordingly, it does not vary significantly after this pertinent date.

The GARCH(1,1) parameters stay practically the same as those obtained prior to adding the dummy variable, which is not surprising given the insignificance of the dummy coefficient for a structural change of volatility after the Brexit referendum.

ARDL Output

In accordance with the existing literature, we implement the ARDL cointegration approach following the steps listed below:

1. Test for unit roots. Although checking for unit roots is not a necessary condition in the ARDL approach (this is a difference with other cointegration approaches), it has been done as an initial step to make sure that none of the variables is I(2) or beyond. If this were the case, ARDL would crash and could yield misleading and/or inconsequential results.

2. Formulate an unrestricted (also called unconstrained, while Pesaran et al. (2001) name it conditional) ECM and determine the appropriate lag structure. In addition, a decision must be made about whether or not to include a time trend in the unrestricted model and whether its coefficient should be restricted. This is decided based on the sample period of the time series under study.

3. Test for the key assumptions of the ARDL bounds testing methodology; namely, Gaussianity, heteroskedasticity and especially serial correlation of the residuals of the model selected in step 2.

4. If the ARDL bounds testing methodology key assumptions are satisfied, test for the dynamic stability of the selected model.

5. In the case of stability, a bounds test aimed at assessing the long-run relationship between the variables is implemented.
6. If the existence of a long-run relationship between the variables is not rejected, estimate a long-run levels model and a separate restricted ECM.
7. Using the estimates obtained in step 6, measure the short-and long-run relationship between the variables.

Unit Root Tests

The existence of a unit root, as is widely recognized, suggests that the time series under examination is non-stationary, which might result in false inferences or erroneous regression. Since cointegration is a highly effective tool for identifying the presence of steady-state equilibrium between variables, it was developed to address the non-stationarity problem (and earlier limitations on the lag structure of a model). When using non-stationary time series data in an economic model, cointegration has come to be viewed as a fundamental necessity. If the variables are not cointegrated, false regression issues occur, and the results lose their significance (Nkoro & Uko, 2016).

In our case study, all the variables have been tested for stationarity using Phillips-Perron (PP) unit root tests including a drift term. Results in Table 5 confirm that all the series are I(1), with the exception of exports (LEX) and exchange rate volatility (LVOL), which are I(0). In other words, the variables included in the export demand function are a mixture of I(0) and I(1), suggesting the suitability of using the ARDL approach for examining relationships in the levels of such variables (see subsection 2.3).

ARDL Optimal Lag Length

The decision about the optimal lag order of the error correction version of the ARDL is usually based on the Final Prediction Error (FPE), the AIC, the Schwarz Bayesian information Criterion (SCIC), and the Hannan and Quinn information criterion (HQIC) (Raza et al., 2015). However, the final decision was taken based on the AIC because it provides the maximum number of lags, so that a larger number of candidate models is estimated, consequently minimizing the risk of not selecting the optimal model (Pesaran et al., 2001). The starting number of lags was set at 8, meaning the estimation of 52,488 candidate models. Results show that ARDL (5, 4, 0, 1, 0) is the best model, as it has the minimum negative value of AIC among
the 52,488 candidate models. In other words, the best model includes five lag terms for EX, four for IPI, one for TOT, and no lag for REER and VOL. Based on the optimal lag structure, Equation (12) below is formulated.

\[
\Delta LEXP_t = \alpha_0 + \lambda_1 LEX_{t-1} + \lambda_2 LIPI_{t-1} + \lambda_3 LREER_{t-1} + \lambda_3 LTO T_{t-1} + \\
+ \lambda_4 LVOL_{t-1} + \sum_{i=1}^{l_1} \theta_{2i} \Delta LIPI_{t-i} + \sum_{i=1}^{l_2} \theta_{2i} \Delta LREER_{t-i} + \\
+ \sum_{i=1}^{l_3} \theta_{3i} \Delta LTO T_{t-i} + \sum_{i=1}^{l_4} \theta_{4i} \Delta LVOL_{t-i} + \varepsilon_{it}
\]  

where \( \theta_{2i}, \theta_{3i}, \theta_{4i} \) and \( \lambda_1, \lambda_2, \lambda_3, \lambda_4 \) represent the short- and long-run coefficients, respectively, of the four drivers of the UK export demand (LIPI, LREER, LTOT, and LVOL).

Bounds testing for level relationships (steps 4 and 5)

In order to observe the key assumptions in the bounds testing methodology, we use the Jarque-Bera, Lagrange Multiplier (LM), and the Breusch-Pagan Godfrey tests to inspect the Gaussianity, serial correlation and heteroskedasticity, respectively, of the selected model residuals. Results in Table 6 indicate that residuals are Gaussian, not serially correlated, and homoskedastic at the 1% significance level. Additionally, the cumulative sum of recursive residuals (CUSUM) and cumulative sum of the squared recursive residuals (CUSUMSQ) tests are used to determine whether the long-run coefficients and short-run dynamics are stable. Figure 3 shows that both CUSUM and CUSUMSQ statistics are inside the critical boundaries, indicating that the model is not structurally unstable, and thus confirming that the cointegration relationship between the variables involved in the UK export demand function is plausible.

The results of the bounds testing for the long-run relationship between UK exports to the Eurozone countries and the set of drivers used as regressors are shown in Table 7, and are provided by Eviews version 10. It can be seen that the value of the F-statistic exceeds the upper bounds provided in Narayan (2005) for a 4-regressor ARDL model with an unrestricted intercept and no trend, at the 5% significance level (third, fourth and fifth columns), suggesting that there is not enough empirical evidence against the

---

3 Narayan (2005) argues that because the critical values provided by Pesaran et al. (2001) are based on large sample sizes, they cannot be used for small sample sizes. For small sample sizes, Narayan (2005) offers a set of critical values that range from 30 to 80 observations (our sample size is n=80). However, the value of the F-bounds test statistics also exceeds the
rejection of the null hypothesis (H$_0$: there is no level long-term relationship between IPI, REER, TOT, and VOL in the export equation, irrespective of whether the regressors are I(0) or I(1)). Accordingly, the selected ARDL model is revised using a single dynamic error correction model to identify both long-run and short-run relationships.

**Long-run and short-run relationship (steps 6 and 7)**

Based on the bounds testing results, the long-run equilibrium relationship between the variables involved in the UK export demand function can be meaningfully estimated using Equation (13).

\[
L\text{EX}_t = \alpha_0 + \lambda_1 L\text{EX}_t + \lambda_2 L\text{IPI}_t + \\
+ \lambda_3 L\text{REER}_t + \lambda_3 L\text{TOT}_t + \lambda_4 L\text{VOL}_t + \epsilon_t
\]  

(13)

As can be checked in Table 8, all the drivers of the UK export demand function significantly explain its variability. Specifically, TOT is significant at the 10% significance level, whereas IPI, REER, and VOL are significant at the level of 5%. From the ARDL (5,4,0,1,0), the empirical results of the long-run relationships (Eq. 13) are presented in Table 8. Notably, the exchange rate volatility has a statistically significant negative impact on exports. This output is in line with the theoretical models where, under scenarios of high volatility, risk-averse traders may limit cross-border transactions because they run the risk of incurring unforeseen costs due to exchange rate fluctuation (Arize & Malindretos, 1998; Doğanlar, 2002; Thuy & Thuy, 2019).

As expected, the impact of VOL on UK exports is lower than that of the TOT, the variable with the greatest long-run effect on UK exports. However, it is not much lower than the size of the effect of the IPI on the volume of the UK export demand (although the impact of the IPI is positive), and is greater than the impact of the REER. Therefore, VOL can be considered a non-negligible driver of the long-run dynamics of UK exports. Specifically, applying the exponential transformation to the coefficient listed in Table 8, a 1% increase in VOL is estimated to reduce the demand for UK exports, ceteris paribus, by approximately 0.78% (see Appendix for details). For the sake of comparison, a 1% increase in REER is estimated to reduce UK ex-

---

Narayan (2005) upper critical value for the significance level of 5%. Accordingly, irrespective of whether we use the sets of asymptotic critical values provided by Pesaran et al. (2001) or Narayan (2005), the decision obtained from the F-bounds test is the same.
ports by 0.60%, whereas the same percentage change in IPI and TOT leads to an increase in UK exports of 2.40% and 9.44%,\(^4\) respectively.

In order to determine the speed of adjustment at which the model returns to its equilibrium, we opted to introduce the ECM coefficient as mentioned above, using the long-run normalized estimates as illustrated in Equation (14):

\[
LEX_t = \alpha_0 + \sum_{i=1}^{l_1-1} \theta_{1i} \Delta LIPI_{t-i}^k + \sum_{i=1}^{l_2-1} \theta_{2i} \Delta LREER_{t-i} + \\
+ \sum_{i=1}^{l_3-1} \theta_{3i} \Delta LTOT_{t-i} + \sum_{i=1}^{l_4-1} \theta_{4i} \Delta LVOL_{t-i} + \beta_1 ECM_{t-1} + \epsilon_t,
\]

where ECM\(_{t-1}\) is the error term, which should be negative and significant, indicating how rapidly variables adjust or revert to their long-run equilibrium.

Table 9 reports only the significant results of the short-run dynamic coefficients. Results show that IPI, TOT, and VOL are significant, displaying an impact on UK exports to Eurozone countries. Interestingly, in the short run, the coefficient of VOL is negative and significant, indicating that if the GBP/EUR exchange rate volatility increases, the UK export volume to the Eurozone countries will decrease in the short run. Our results support those of Arize and Malindretos (1998) and Srinivasan and Kalaivani (2013), who argued that higher exchange rate volatility will depress export volume because of an increase in adjustment costs due to higher uncertainty and risks, and that the export volume is reduced due to the lack of hedging opportunities which causes risk-averse firms to reduce their exports.

The short-run coefficient of IPI is positive and significant at \(t-3\), suggesting that trading partners’ real income exerts a positive impact on UK exports in the short run. Surprisingly, both the short-run and long-run coefficients of TOT are positive and significant at 1%. Since we used the Commodity Terms of Trade as a proxy for TOT, the positive sign could be attributed to the fact that the number of country-specific commodities this index considers is limited to 45, which might affect TOT representativeness. It could also be due to the high sensitivity of commodity markets to

\(^4\) These three percentages are approximate figures.
economic, social or political shocks, especially in distress periods. In such a case, an increase in the demand for UK commodities would be compatible with an increase in export prices. Finally, an increase in REER does not show any significance in the short run. However, the REER coefficient is negative and statistically significant at 5% in the long run, implying that an appreciation in the value of the pound relative to the euro will reduce the competitiveness of UK goods and services in the Eurozone market in the long run.

The ECM coefficient is -0.4745 and is statistically significant at the 1% significance level. The negative sign indicates the presence of disequilibrium in earlier short-run periods and is further evidence of cointegration among the variables involved in the UK export function. As outlined above, the value of 0.4745, which indicates the speed of adjustment, suggests that 47.45% of the deviation from the long-run equilibrium period between variables is periodically corrected.

Finally, and in order to assess the stability of the parameters, we have also implemented the CUSUM and CUSUMQ tests. The results obtained indicate the absence of instability. The serial correlation, heteroskedasticity, and normality tests applied to the ECM equation shown in Table 10, reveal satisfactory outcomes.

Discussion

Following the Brexit referendum vote, which created a significant amount of ambiguity and uncertainty regarding the future of the UK economy, this paper aims to provide insight and clarity on the effects of Brexit on the volatility of the GBP/EUR exchange rate and on UK exports to Eurozone countries. By examining these specific economic factors, the paper seeks to shed light on the potential impact of Brexit and help alleviate some of the uncertainty surrounding the issue. The results showed that GARCH (1,1) was the best model for capturing the volatility of the exchange rate both before and after the Brexit vote. However, when we used the same GARCH (1,1) model but incorporated a dummy variable to examine whether the volatility structure of the exchange rate had changed after the Brexit vote, the results did not show any significant evidence of such a change. This suggests that while the GARCH (1,1) model was effective in capturing the
overall volatility of the exchange rate, the Brexit vote did not have a significant impact on the volatility structure of the exchange rate.

Also, the application of the ARDL bounds testing approach to the UK export demand function to examine the relationship between the volume of UK exports and its main determinants — specifically the Eurozone industrial production index, the real effective GBP/EUR exchange rate, the terms of trade, and the GBP/EUR exchange rate volatility — revealed a cointegration relationship between UK exports and these main drivers, which suggests that these variables are influenced by each other in the long run. The study also found that there is fairly rapid adjustment to the long-run equilibrium, meaning that the UK export demand function is quick to react and adjust to changes in these key drivers. In particular, it was found that the industrial production index had a positive impact on the volume of UK exports in both the long and the short run, which indicates that as the industrial production in the Eurozone increases, the demand for UK exports also increases. This finding highlights the importance of the industrial production index as a determinant of UK exports and the strong interdependence between the UK economy and the Eurozone economy. It is an interrelationship rooted in the close trade, investment, and financial ties between the UK and the countries in the Eurozone, where changes in one economy can have significant impacts on the other. As the UK is one of the largest trading partners of the Eurozone countries, and given the significance of its financial sector and its role as a gateway to the rest of the world, any changes in the UK economy can also affect the Eurozone’s economy. Additionally, the close relationship between the UK and the Eurozone is reflected in the fact that the British pound and the euro are among the most widely traded currencies in the world. Changes in the exchange rate between these two currencies can have significant impacts on the economies of both the UK and the Eurozone.

The findings of our study make a useful contribution to the growing body of research suggesting that exchange rate volatility has a significant impact on exports. Our results partially support the findings of Smallwood (2019), Sugiharti et al. (2020), and Bahmani-Oskooee and Saha (2021), who conducted similar research and found evidence of a negative relationship between exchange rate volatility and exports. Furthermore, our results align with the conclusion of Hurley and Papanikolaou (2021), who confirmed the short- and long-run impact of exchange rate volatility on exports. This highlights that the negative effects of exchange rate volatility
not only emerge immediately but also persist over time. This is important information for policymakers and businesses alike as it underscores the need for measures to mitigate the negative effects of exchange rate volatility on exports.

Moreover, this paper offers new empirical evidence of the long-run impact of exchange rate volatility on the UK’s export performance. Specifically, a 1% increase in exchange rate volatility (VOL) would reduce the demand for UK exports by 0.78%. Having this knowledge can help businesses to better understand the risks associated with exchange rate fluctuations, which is critical as UK exports to the Eurozone decreased significantly in 2020 and only slightly recovered in 2021. A combination of factors likely accounts for this drop, including the COVID-19 pandemic and the uncertainty surrounding the Brexit process, which added to the challenges already faced by the UK economy. The fact that UK exports to the Eurozone recovered only slightly in 2021 is likely due to continued uncertainty surrounding Brexit, as well as other ongoing challenges; specifically, those related to supply chain disruptions, as well as changes in consumer behavior and preferences.

Finally, the use of the ARDL approach provides robust and reliable evidence of the impact of exchange rate movements on the competitiveness of UK exports in the international market. Our results confirm that an appreciation in the value of the pound relative to the euro has a negative impact on the competitiveness of UK exports in the long run, with no significance in the short run. This suggests that the impact of exchange rate movements on UK exports takes time to materialize.

Overall, the interdependence between the UK economy and the Eurozone economy reflects their close and complex relationship and points to the need for cooperation and coordination in order to promote stability and growth in both regions.

Conclusion

In conclusion, this study provides a comprehensive analysis of the economic implications of Brexit for the UK and the Eurozone, particularly in terms of exchange rate volatility and its effect on UK exports. The results indicate that exchange rate volatility has a detrimental impact on export volume in both the short and the long run, while the industrial production index has
a positive effect. Additionally, the appreciation of the pound relative to the euro also adversely affects the competitiveness of UK exports in the long run. This study offers valuable insights into the potential consequences of a European member country leaving the EU and serves as a benchmark for future research in this area.

The findings of this study have far-reaching implications for policymakers and businesses alike. It is clear that exchange rate volatility has a significant impact on UK exports to the Eurozone, thus emphasizing the need for stability in the currency market if businesses are to remain competitive. In light of this, policymakers should prioritize addressing exchange rate volatility as a key factor in sustaining trade relationships with the Eurozone. By stabilizing the currency market, policymakers can create a more conducive environment for businesses to export their products and mitigate the adverse effects of exchange rate volatility on trade. For businesses, the results of this study emphasize the importance of closely monitoring exchange rate volatility and implementing effective hedging strategies to minimize its impact on their exports. This will enable businesses to remain competitive and continue to expand their trade relationships with the Eurozone.

Further research is necessary to confirm these findings and to provide a more comprehensive understanding of the economic implications of Brexit and other similar events. This could include investigating the impact of exchange rate volatility on UK exports to other major trading partners, examining the asymmetrical impact of exchange rate volatility on exports using higher-frequency data, and exploring the long-term effects of Brexit on the UK economy and trade relationships. Furthermore, using industry-specific and country-specific data to study the impact of exchange rate volatility on bilateral trade between the UK and its partners, and within each industry, will yield more detailed and accurate results and reduce aggregation bias. Also, a more comprehensive understanding of the economic implications of Brexit can be gained by examining the long-term effects of Brexit on the UK economy, considering ongoing negotiations and future trade agreements. Similarly, it would be worth investigating how a future exit by other countries from the EU could potentially influence their trade relationships and exchange rate volatility.

Despite the thorough analysis and rigorous methods used in this study, there are still some limitations that need to be acknowledged. The study focuses on the UK-Eurozone trade relationship and the impact that Brexit has had on it, but does not consider the potential impact of Brexit on the
UK’s trade relationships with other countries or regions. Additionally, the analysis only covers the period from January 1, 2010 to August 31, 2020, which may not be sufficient to gain a complete picture of the long-term effects of Brexit. Lastly, accounting for other factors that may affect trade between the two regions, such as trade agreements, tariffs, and non-tariff barriers, could provide more insights into the UK-Eurozone trade relationship.

References


Annex

Table 1. Summary statistics of variables

<table>
<thead>
<tr>
<th>Variables</th>
<th>Mean</th>
<th>Median</th>
<th>Std. Dev.</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Jarque-Bera</th>
<th>P-value</th>
<th>No. Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: Daily Data Jan 1, 2010-Aug 31, 2020</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Return</td>
<td>1.30e-0.5</td>
<td>0.0000</td>
<td>0.00524</td>
<td>-0.36839</td>
<td>8.055</td>
<td>3024.4</td>
<td>0.00</td>
<td>2781</td>
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<tr>
<td>Panel B: Monthly Data Jan 2010- Aug 2020</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LEX</td>
<td>9.678</td>
<td>9.699</td>
<td>0.120</td>
<td>-0.907</td>
<td>4.784</td>
<td>34.529</td>
<td>0.000</td>
<td>128</td>
</tr>
<tr>
<td>LIPI</td>
<td>4.656</td>
<td>4.641</td>
<td>0.056</td>
<td>-0.288</td>
<td>3.267</td>
<td>2.146</td>
<td>0.342</td>
<td>128</td>
</tr>
<tr>
<td>LREER</td>
<td>4.626</td>
<td>4.608</td>
<td>0.051</td>
<td>0.898</td>
<td>2.913</td>
<td>17.250</td>
<td>0.000</td>
<td>128</td>
</tr>
<tr>
<td>LTOT</td>
<td>4.587</td>
<td>4.588</td>
<td>0.016</td>
<td>-0.208</td>
<td>2.054</td>
<td>5.698</td>
<td>0.058</td>
<td>128</td>
</tr>
<tr>
<td>LVOL</td>
<td>0.000</td>
<td>-0.001</td>
<td>0.022</td>
<td>-0.267</td>
<td>3.446</td>
<td>2.565</td>
<td>0.277</td>
<td>128</td>
</tr>
</tbody>
</table>

Table 2. EWMA, GARCH(1,1) and EGARCH(1,1) parameter estimates

<table>
<thead>
<tr>
<th></th>
<th>EWMA</th>
<th>GARCH(1,1)</th>
<th>Prob.</th>
<th>EGARCH</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>λ</td>
<td>0.95341</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ω</td>
<td>0.00001</td>
<td>0.0000</td>
<td>-0.38471</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>α</td>
<td>0.06724</td>
<td>0.0000</td>
<td>0.15012</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>β</td>
<td>0.91600</td>
<td>0.0000</td>
<td>0.97446</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>α + β</td>
<td>0.98320</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>γ</td>
<td>-0.02828</td>
<td></td>
<td>0.0490</td>
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<td></td>
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<tr>
<td>Log Likelihood value</td>
<td>26663.56</td>
<td>10799.60</td>
<td>6889.58</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3. Error statistics

<table>
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<tr>
<th>Models</th>
<th>RMSE</th>
<th>Rank</th>
<th>MAE</th>
<th>Rank</th>
<th>MAPE</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>EWMA</td>
<td>0.030731</td>
<td>2</td>
<td>0.022848</td>
<td>2</td>
<td>36.301547</td>
<td>2</td>
</tr>
<tr>
<td>GARCH(1,1)</td>
<td>0.030548</td>
<td>1</td>
<td>0.022649</td>
<td>1</td>
<td>37.703237</td>
<td>3</td>
</tr>
<tr>
<td>EGARCH(1,1)</td>
<td>0.038319</td>
<td>3</td>
<td>0.028337</td>
<td>3</td>
<td>34.536369</td>
<td>1</td>
</tr>
</tbody>
</table>

\[
\text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} e_i^2} ; \quad \text{MAE} = \frac{1}{n} \sum_{i=1}^{n} |e_i| ; \quad \text{MAPE} = \frac{1}{n} \sum_{i=1}^{n} \frac{|e_i|}{VR_i},
\]

where \( VR_i \) is the realized volatility.
Table 4. Parameter estimates of the GARCH(1,1) with a dummy variable

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Coefficients</th>
<th>p-values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega$</td>
<td>4.95E-07</td>
<td>0.0001</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.067254</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.916144</td>
<td>0.0000</td>
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<tr>
<td>$\varphi$</td>
<td>2.74E-08</td>
<td>0.7113</td>
</tr>
</tbody>
</table>

*Log likelihood value = 6890.238

Table 5. Phillips-Perron unit root tests

<table>
<thead>
<tr>
<th>Levels</th>
<th>Levels</th>
<th>First Difference</th>
<th>First Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Constant</td>
<td>p-value</td>
<td>Constant &amp; Trend</td>
</tr>
<tr>
<td>LEX</td>
<td>-5.6035***</td>
<td>0.0000</td>
<td>-6.3903***</td>
</tr>
<tr>
<td>LIPI</td>
<td>-2.2924</td>
<td>0.1760</td>
<td>-2.1592</td>
</tr>
<tr>
<td>LREER</td>
<td>-1.792541</td>
<td>0.3828</td>
<td>-1.8566</td>
</tr>
<tr>
<td>LTOT</td>
<td>-2.0815</td>
<td>0.2525</td>
<td>-2.4570</td>
</tr>
<tr>
<td>LVOL</td>
<td>-5.4769***</td>
<td>0.0000</td>
<td>-5.4048***</td>
</tr>
</tbody>
</table>

Note: *** indicates significance at the 1% significance level.

Table 6. Diagnostic tests

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Gaussianity Test: Jarque-Bera</th>
<th>Breusch-Godfrey Serial Correlation LM Test:</th>
<th>Heteroskedasticity Test: Breusch-Pagan-Godfrey</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>Jarque-Bera statistic: 1.220171</td>
<td>F-statistic: 0.878675</td>
<td>F-statistic: 0.827376</td>
</tr>
<tr>
<td>Distribution</td>
<td>Chi-Square (2)</td>
<td>F (8, 37)</td>
<td>F (24,45)</td>
</tr>
<tr>
<td>p-value</td>
<td>0.5433</td>
<td>0.5373</td>
<td>0.6455</td>
</tr>
<tr>
<td>Obs*R-squared</td>
<td>8.154493</td>
<td>Obs*R-squared: 12.78370</td>
<td>Obs*R-squared: 12.36613</td>
</tr>
<tr>
<td>Distribution</td>
<td>Chi-Square (8)</td>
<td>Chi-Square (24)</td>
<td>Chi-Square (24)</td>
</tr>
<tr>
<td>p-value</td>
<td>0.4185</td>
<td>0.6190</td>
<td>0.9476</td>
</tr>
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</table>
Table 7. F-bounds test statistics to test the existence of long run relationships

<table>
<thead>
<tr>
<th>F-bounds test</th>
<th>Null hypothesis: No levels relationship</th>
<th>Test statistic</th>
<th>Value</th>
<th>Significance</th>
<th>I (0)</th>
<th>I (1)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>F-statistic</td>
<td>3.8020</td>
<td>10.00%</td>
<td>2.303</td>
<td>3.220</td>
</tr>
<tr>
<td></td>
<td></td>
<td>k (number of regressors)</td>
<td>4</td>
<td>5.00%</td>
<td>2.688</td>
<td>3.698</td>
</tr>
</tbody>
</table>

Table 8. Long-run coefficients estimates of the linear ARDL model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIPI</td>
<td>0.8723</td>
<td>0.3402</td>
<td>2.5640</td>
<td>0.0117**</td>
</tr>
<tr>
<td>LREER</td>
<td>-0.5347</td>
<td>0.2566</td>
<td>-2.0838</td>
<td>0.0396**</td>
</tr>
<tr>
<td>LTOT</td>
<td>2.2448</td>
<td>1.2952</td>
<td>1.7333</td>
<td>0.0856*</td>
</tr>
<tr>
<td>LVOL</td>
<td>-0.2458</td>
<td>0.1157</td>
<td>-2.1240</td>
<td>0.0360**</td>
</tr>
<tr>
<td>@TREND</td>
<td>-0.0019</td>
<td>0.0005</td>
<td>-3.9364</td>
<td>0.0001***</td>
</tr>
</tbody>
</table>

Note: *, **, and *** indicate significance at the 10%, 5%, and 1% levels of significance, respectively

LEX_t = −0.0019 + 0.8723 LIPI_t − 0.5347 LREER_t + 2.2448 LTOT_t − 0.2458 LVOL_t + ε_t

Table 9. ECM for the selected ARDL short-run coefficients estimates of ARDL model (Eq. 14)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ΔLEX_{t-4}</td>
<td>0.3257</td>
<td>0.0863</td>
<td>3.7725</td>
<td>0.0003***</td>
</tr>
<tr>
<td>ΔLIPI_{t-3}</td>
<td>0.6458</td>
<td>0.3037</td>
<td>2.1266</td>
<td>0.0357**</td>
</tr>
<tr>
<td>ΔLTOT</td>
<td>6.0892</td>
<td>1.2545</td>
<td>4.8539</td>
<td>0.0000*</td>
</tr>
<tr>
<td>ΔLVOL</td>
<td>-0.2180</td>
<td>0.0938</td>
<td>-2.3243</td>
<td>0.0219**</td>
</tr>
<tr>
<td>ECM_{t-1}</td>
<td>-0.4745</td>
<td>0.1095</td>
<td>-4.3348</td>
<td>0.0000*</td>
</tr>
</tbody>
</table>

P-value (F-statistic) | 0.0000 | CUSUM | Stable |
Adjusted R-squared | 0.5431 | CUSUMSQ | Stable |

Note: *, **, and *** indicate significance at the 10%, 5%, and 1% levels of significance, respectively.
Table 10. Diagnostic tests of the ECM equation

<table>
<thead>
<tr>
<th>P-value</th>
<th>Value</th>
<th>Distribution</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Breusch-Godfrey Serial Correlation LM Test</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F-statistic</td>
<td>1.022614</td>
<td>F (8,92)</td>
<td>0.4240</td>
</tr>
<tr>
<td>Obs*R-squared</td>
<td>9.132724</td>
<td>Chi-Square (8)</td>
<td>0.3312</td>
</tr>
<tr>
<td>Heteroskedasticity Test: Breusch-Pagan-Godfrey</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F-statistic</td>
<td>0.563401</td>
<td>F (19,100)</td>
<td>0.8671</td>
</tr>
<tr>
<td>Obs*R-squared</td>
<td>7.122075</td>
<td>Chi-Square (19)</td>
<td>0.8494</td>
</tr>
<tr>
<td>Scaled explained SS</td>
<td>4.374252</td>
<td>Chi-Square (19)</td>
<td>0.9757</td>
</tr>
<tr>
<td>Normality Test: Jarque-Bera</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>1.319117</td>
<td>Chi-Square (2)</td>
<td>0.5171</td>
</tr>
</tbody>
</table>

Figure 1. Time series plot of the daily GBP/EUR exchange rate simple returns between Jan 1, 2010 and Aug 31, 2020
Figure 2. Realized volatility vs. volatility estimated with the competing models

Figure 3. Stability tests
Figure 3. Continued