ORIGINAL PAPER

Citation: Vukovic, D. B., & Prosin, V. (2018). The prospective low risk hedge fund capital allocation line model: evidence from the debt market. *Oeconomia Copernicana*, 9(3), 419–439. doi: 10.24136/oc.2018.021

Contact to corresponding author: vdarko@hotmail.rs; National Research University Higher School of Economics, St. Petersburg School of Economics and Management, Department of Finance, Kantemirovskaya 3A, St. Petersburg, Russia.

Received: 20 June 2018; Revised: 8 August 2018; Accepted: 19 August 2018

Darko B. Vukovic

National Research University Higher School of Economics

National Research University – South Ural State University, Russia

Victor Prosin

National Research University Higher School of Economics, Russia

The prospective low risk hedge fund capital allocation line model: evidence from the debt market

JEL Classification: G11; G12; G23

Keywords: *portfolio; Sharpe ratio; risks; hedge fund; capital allocation line model*

Abstract

Research background: Institutional investors such as: commercial banks, pension funds, and insurance companies are constantly looking for low-risk stable investment opportunities, whereas one of the solutions can be a simulated portfolio. This research takes a look at the incentive to invest in government debt portfolios, as it can outperform the returns of deposit accounts.

Purpose of the article: This study considers several classic methods of portfolio construction and includes the basis of debt instruments that have not been a research topic for a long period of time. At the same time, this paper analyzes the classic methods of modern portfolio theory with a Sharpe ratio as an indicator of efficiency.

Methods: The constructed portfolio consists of four elements from different countries: two government obligations and two bond indexes, aiming to employ international diversification. All the data was collected for the period of 12 years in order to represent the consequences of accrued recessions.

Findings & Value added: The past two severe financial crises created a higher demand for stable investments, and more investors are ready to compromise a higher return for it. Therefore, the results of this paper represent a simulation of low-risk hedge fund portfolio construction with the use of highly rated debt instruments.

Introduction

The institutional players use several tools or combinations of them in order to construct efficient portfolio, whereas all of those tools have a number of constraints with respect to risk-return trade-off. Hedge funds can be a good example of investment institutions that are constantly evaluating their investment analysis methods, making it an ideal choice for this research. Majority of the available studies are focused on the stock portfolios analysis and not on the full debt market portfolios, when pension funds and insurance companies greatly contribute to the overall investment in the low-risk debt instruments, due to the imposed government regulations. In order to analyse the topic more deeply, it is necessarily to evaluate the current conditions of the debt instrument portfolios. In the past, there have been a vast number of studies that concerned the field of portfolio construction, but a large part of it was focused on high-risk assets. The current market conditions have been negatively affecting the debt market, making it crucial to evaluate the conditions, and to provide an approximation of the possible risk levels, or expected returns achieved from the low-risk portfolios that hedge funds are using. In the period of economic instability across the world, it is vital to evaluate long-term stable investments, which are of low-risk nature. In the current paper, the constructed portfolio is a simulation of a low-risk hedge fund portfolio that requires long-term stable investments on the basis of two index funds, and two government obligations graded AAA by the rating agencies. The result provides an analysis of the historical fluctuations in bond yields and forecasts future expected return on a similar risk level of investment.

Today, as a result of the uncertainties in the economic environment, the risk-averse investors are trying to minimize the possibility of losing the invested capital. That is the reason for the higher demand of the low-risk investment opportunities with a high risk premium-to-volatility ratio and a constant stable return for the whole period of maturity ensuring the profitability of the investment. Today, savings accounts in banks worldwide provide a very low return rate, which can be outperformed by international bond portfolio; in Russia the interest rates on savings accounts in Euro and USD are 0.1% on average, a Euro deposit in Germany brings 0.75%, in the United States the USD account would bring 1.7% and a Canadian dollar

account could bring 0.25% (Deposits.org, 2018). Taking into consideration that those are only annual rates, which do not reflect the continuous compounding, a comparison of the return on a savings account and the yield-to-maturity (YTM) of a low risk debt instrument will show the advantages of low risk portfolio investments, for the same time period.

Hedge funds (HF) generally stick with a long position of buy-and-hold on regular assets, capturing the risk premium of the equity risk, interest rate risk and default risk, making it incompetent to capture the benefits of the dynamic trading strategy (Agarwal & Naik, 2003). The low-risk HF is primarily concerned with investment in risk-free or close to risk-free securities such as bonds, T-bills and Mortgage Backed Securities (MBS). Each HF has its own allocation optimization system to provide the best possible performance for its customers (taking a charge for it), where the need of portfolio optimization comes into place with the prediction of the future behaviour of assets. Nowadays, it is crucial to understand that the optimal portfolio of hedge funds is not just simply based on the mean-variance analysis.

This research aims to analyse the low-risk HF portfolio asset allocation methods and construct an optimal portfolio. In this research, we used a simulation of a portfolio that consists of different 10-year government bonds. After calculations of expected returns, portfolio standard deviation and covariance, the mean-variance model for the portfolio optimization was applied. Finally, we calculated maximized Sharpe ratio coefficient for the efficient investment allocation.

The paper is divided into the following parts: the literature review that will be covering all the relevant world literature that can be relevant to the study, and the second part is methodology, which will provide us the methods applied for the analysis. The final part of this research will be the discussion and conclusion.

Literature review

The different nature of styles applied for this research provided us with several limitations, specifically for theories comparison that are used in the papers. As the time horizon, size of the used sample and the methodology changes all vary from each other, it only allowing the results to be compared on the one-to-one basis, however, all of the variations have to be considered when analysing the results. The topics of forecasting, the market movements and the different financial instruments behaviour are always an interesting matter for research since they provide the grounds for the analysis of different asset performance scenarios. They provide new and accurate

methods of earning high returns and make the topic important for the professionals, generally boosting the popularity in the field of research (Majhi *et al.*, 2009). All of the predictions are based on the assumption that the fundamental information can influence the publicly available stock prices, which while can be exploited for the future price movement's predictions. However, this contradicts the efficient market hypothesis that the current market prices includes all the information available, and no one can take advantages of the asymmetric information, making it impossible to predict the future behaviour of stock. The evidence available in the sphere of research shows that market does not follow the perfect world scenario, and the prices do not just follow the random walk pattern being an asymmetric fat-tailed distribution; decreasing the possibility for the future prediction of the stocks (Lo & MacKinlay, 1988; Vykyuk *et al.*, 2013). The implied market expectation is the key to investor's decisions, at the point where personal expectations diverge from the market ones, and it is the key to adjusting the contents of portfolio to take advantage of what is known as market error (Barr & Campbell, 1997; Meluzín *et al.*, 2017; 2018a; 2018b; Szumilo *et al.*, 2018).

The risk plays a very important role in the sphere of investment. In the case without any risk, there will be no possibility for the returns to differ, and everyone would get the same ones. Therefore, the risk is a key determinant in the investment strategy formulation, financial decisions and economic activity (Machina & Rothschild, 2008; Fraś, 2018). There is a significant relation between the expected return and stock volatility. In the case of higher volatility (risk), the expected return is higher. This relation is commonly measured as a covariance between the future possible return and standard deviation (French *et al.*, 1987). The risk is justified by the utility function of investor — one would always prefer to have an expected return $E(x^*)$ for a given risk x^* ; if the investor is risk-averse, than the utility function has to be concave (Machina & Rothschild, 2008). The combination of assets chosen by the investor is combined into a group called portfolio. The interrelations between all of the portfolio components, such as covariance between each asset, expected return and volatility contributes to the total parameters of a portfolio performance. It can be represented as all possible combinations of weights of assets in a portfolio on an investment opportunity set (efficient frontier) (Bodie *et al.*, 2013).

In the model, the tangent point between the capital market line and the efficient frontier is the market portfolio. The CAPM and arbitrage pricing theory provide a basis for the most of asset valuation literature, which generally deals only with linear payoffs. For the bonds that are bought today the measure of the return is a yield to maturity, it has to be constantly re-

vised, as bonds have a fixed maturity period. In any case, the investor has to choose between the “risk-free” and risky investment, taking into account that risk-free investment is not actually completely safe, as risk in any investment is inevitable. A portfolio owner can take any possible point inside (or on the investment possibility set for diversification of assets in portfolio), taking into consideration that for higher returns investor has to bear higher risks. Another very useful measurement of the risk-return trade-off is the Sharpe coefficient, which measures the relations between each unit of standard deviation and extra return that is rewarded for the given change in risk (Bodie *et al.*, 2013).

At the rise of the hedge funds (HF) era they have been bombarded by the investors all over the globe, for its ability to secure the investments against extraordinary situations on the market by hedging the positions, effectively reducing risk (Lhabitant, 2001). Nowadays, they are very similar to mutual funds that allow private investors to pool funds for hedge fund managers to invest. Such funds are generally organized as private partnerships in order to reduce SEC regulations, and they mainly work with wealthy/institutional investors (Bodie *et al.*, 2013). Hedge funds generally stick with long position of buy-and-hold on regular assets, capturing the risk premium of equity risk, interest rate risk and default risk, making it incompetent to capture the benefits of dynamic trading strategy (Agarwal & Naik, 2003). The low risk HF is primarily concerned with the investment of risk-free or close to risk free securities such as bonds, t-bills and MBS (Bodie *et al.*, 2013). Each HF has its allocation optimization to provide the best possible performance for its customers (taking the charge for it), where the need of portfolio optimization comes into place with prediction of future behaviour of assets. Nowadays, it is crucial to understand that the optimal portfolio of hedge funds is not just simply based on mean variance analysis. Brooks and Kat (2002) revealed the strategy of hedge funds, which showed that the returns are negatively skewed and have kurtosis level which is higher than the normal distribution. There are a great number of methods and risk measurements that are accepted in the sphere of hedge fund investment; however, none of the techniques have emerged into a unique set of rules (Lamm, 2003). In addition, Lamm in his analysis stated that the low-risk-hedge funds portfolios could include a big fraction of neutral equities, convertible arbitrage and merger arbitrage. Amenc and Martellini (2002) reported from the analysis of European multi-managers that only 2% take into account skewness and kurtosis, 13% implement extreme risk measure and scenario planning; where the predominantly used measures are volatility (84%) and Sharpe ratio (82%) (2002).

From the start of 1990, the markets of capital around the world have been growing and experienced a particular interest from the side of public participation on the bond markets. Governments' presence on the world-wide markets have been growing sharply (Claessens *et al.*, 2007, Vukovic *et al.*, 2017). There are many different impacts that affect bond prices and one of the key impacts is the interest rate. In the work of Bernoth *et al.*, (2004) it has been shown that in a portfolio consisting of 2 bonds (domestic and international) there are three main risks associated with such an investment: default risk premium positively related to the probability of default by the issuer, liquidity risk premium negatively correlated with the liquidity of the local debt market, and thirdly, the country-specific risk premium unique to each country as well as related to the total domestic supply of bonds relative to the total wealth. One of the classic measures of bond payment inflows is Macaulay duration represented as a weighted average of repayments of a zero coupon bond with consecutive repayments as a coupon, and a final large repayment equal to a face value (Hopewell & Kaufman, 2017). Where the value of duration is regarded as an average lifetime of bond and it is equals to a maturity of a zero coupon bond located on the market at the same time and price, providing the same yield; as well as, obtaining a par value of equivalent to the summation of coupons being reinvested till the last day of maturity (Hopewell & Kaufman, 2017). Fisher, Hopewell and Kaufman showed that differential of a bond price with respect to YTM is linear to Macaulay's duration, which shows that for a unit change in IR the change in price of bond will be larger: the lower the coupon rate, the lower the market yield and the longer the maturity (Bierwag *et al.*, 1978). Duration is inversely related to the market yield, coupon rate and yield to maturity (Hawawini, 1984). However, yield to maturity is commonly used in the portfolio calculation for the expected return, and duration is used for the measure of the interest rate risk affecting the particular bond. The YTM is calculated on the basis of coupon rate, face value and its current market price; it represents investor gain if the bond is held until the maturity (Caks, 1977).

Diversification is one of the most important concepts in the world of finance, which is famously known as "don't put all eggs in one basket". Mean-variance portfolio is dealing with investment in several securities to obtain a trade-off between risk and return of types of assets (Lim & Zhou, 2002). The founder of the theory is Harry Markowitz (1952). Markowitz proposed a variance as the risk measure of portfolio in the problem of risk-return-trade-of and where the portfolio returns should be taken as the average weighted of returns. The risk of portfolio is dependent on the covariance between assets included. The objection of portfolio is to maximise

return at the same time by the means of diversification (preferably having correlation between assets close to -1) lower down the risk of overall portfolio. Markowitz theory provides us with a set of optimal portfolios located on the efficient frontier, which maximises risk for a given variance, providing a lowest variance for its given level of risk and the highest return for its variance. Even though bonds are regarded as a “risk free” investment, this is not true in the real life situation; it is subject to risks factors, which have a significant influence on the bond investments. The crucial significance to a fixed income security with a long term fixed maturity has an interest rate (IR) risk (in the era of constantly changing interest rates) and an investor could attract an effect of better investment opportunities if the IR is going up. Fixed income securities provide a constant nominal value return, where if the IR is going up, alternative investments appear more attractive and the PV of cash flows fall (Benczur, 2001). Diversification can be effectively improved, if the investor chooses to expand to international markets, as the correlation between different countries can be negative; however, this can lead to the currency-exchange risk to arise. The value of the investment can significantly decrease due to the fluctuations in the currency exchange between the initial investment currency (currency of the country where investor is located) and the foreign currency of the asset invested in. The main tool of eliminating such an exposure is to hedge the currency exchange rate, but the transaction costs in some circumstances can outperform the possible losses of currency fluctuations (Santis & Gerard, 1998). One of the other key risks associated with debt securities is a default risk, which could arise from the asset underlying the security to pay off less than it is promised, or the issuer of the security to default, making it impossible to pay out its obligation (Merton, 1974). Bonds that are highly graded by the rating agencies are assumed to be risk-free from default, despite the fact that there is still a small possibility of default.

Research methodology

This research analyses portfolio of low-risk hedge fund. The portfolio consists entirely of debt market instruments, such as government bonds of different countries. The low-risk long only portfolio in the US has on average 0.5 Sharpe ratio and for the other markets this value growth, but not significantly (Chow *et al.*, 2014). Diversification is key for an efficient portfolio and only this criterion can minimise unsystematic risk arising from investments on the market. In our portfolio, we used historical yearly data, taken for the past 10 years on bond yields, expected yield of each bonds and

standard deviation of returns. Despite the nature of the US market, international diversification can increase the Sharpe ratio, keeping the risk approximately equal to the US market.

Modern portfolio theory construction method is used to provide the analytics of the portfolio. The model is well known and widely used throughout the investment institutions nowadays.

Throughout the paper following calculations are used (Bodie *et al.*, 2013):

Portfolio expected return:

$$E(r_p) = \sum_i^N w_i E(r_i) \quad (1)$$

where $i=[1;N]$ and denotes the number of assets in portfolio, w_i represents the weight of portfolio invested in particular asset and $E(r_i)$ represents expected return based on the historical performance.

Portfolio variance:

$$\sigma_p^2 = \sum_i \sum_j \sigma_{ij} = \sum_i w_i^2 \sigma_i^2 + \sum_i \sum_{j \neq i} w_i w_j \sigma_i \sigma_j \rho_{ij} \quad (2)$$

where, σ_{ij} is sample covariance of historical returns on an asset, ρ_{ij} is correlation coefficient between two returns of i and j and σ_i is a standard deviation of the return. Where the covariance (Cov) matrix $N \times N$ is used with the diagonal entries being variance of each asset included in the portfolio. Each entry represents the covariance of the element on the vertical with respect to the horizontal entry. Where σ_{1N} represents the covariance of the return of first asset with respect to the return of the asset numbered N .

$$\begin{pmatrix} \sigma_1^2 & \cdots & \sigma_{1N} \\ \vdots & \ddots & \vdots \\ \sigma_{1N} & \cdots & \sigma_N^2 \end{pmatrix} \quad (3)$$

The covariance is used to represent the changes in one variable, if there has been a change in the other variable, if the $\text{Cov} > 0$ then the returns move in the same direction, and if $\text{Cov} < 0$ then the returns move in opposite directions. Covariance shows the effect that each asset brings with respect to other assets to portfolio diversification. Covariance is also expressed as:

$$\sigma_{ij} = \sigma_i \sigma_j \rho_{ij} = E[(w_i - E(w_i))(w_j - E(w_j))] \quad (4)$$

where the correlation coefficient represents the relationship between two variables. If it is positive, then both returns move in the same direction, where negative correlation represents movement in opposite directions if there is a change in one of the assets. While the zero correlation represents no relationship, if there is a change in one asset there is no change in the other one.

Markowitz (1952) is the founder of mean variance analysis, first to state that the portfolio allocation has to consider mean and variance of the assets included. Providing the proof of what is known nowadays as investment opportunity set, where for each given level of standard deviation we consider maximising the expected return and *vice versa*. The investor can choose any point on the curve depending on its risk tolerance. Markowitz was the first to introduce the notion of taking into consideration not just the characteristic of each individual asset, but to include the correlation between the movements of assets. This has started the diversification strategy that including items in portfolio with negative correlation can decrease the overall standard deviation providing the same level of return. The mean-variance model is widely used even today in all institutions for a static portfolio optimization. According to Giamouridis and Vrontos (2005), hedge funds are under constant research by the scientist to analyse the optimization methods and the allocation ratios for the investment. Excel solver is used for the optimization to plot of efficient frontier with constraints provided below.

$$\text{Objective: MAX } E(r_p) \text{ subject to } \sigma_p^2 = C \quad (5)$$

The aim is to maximise the expected return of the portfolio by adjusting the weights of each asset included. Where the efficient frontier represents a set of points with maximum expected return for given level of risk, representing all efficient allocation of resources with regard to the assets included. The capital allocation line, which is tangent to the investment opportunity set at the point at which the Sharpe ratio coefficient is maximised. At this point, the highest reward to volatility ratio is located, making the highest return being accepted for each unit of risk taken (Bodie *et al.*, 2013):

$$S = \frac{\text{Portfolio risk premium}}{\text{Standard deviation of portfolio}} = \frac{E(R_p) - r_f}{\sigma_p} \quad (6)$$

$$E(R_p) = \text{return of portfolio}, \quad (7)$$

where, r_f is risk free rate.

Sharpe ratio is one of main tools for evaluating efficiency of portfolio, taking a risk premium of the portfolio with relation to the risk (Sharpe, 1966). Even if it is assumed that the returns of hedge funds are not normally distributed and make Sharpe ratio inappropriate to use, Martin *et al.* (2006) carried out a research on the different performance metrics on hedge funds returns proving that the Share ratio is a suitable metric, proving a similar ranking compared to other KPIs.

The following constraints are used in the optimization problem. The weights of the assets cannot be negative, making short selling impossible and implying only long positions. In the portfolio similar to low-risk hedge funds which is constructed, only debt instruments are used, for which it is not natural to short sell (even though some of the investors use this option), and short selling increases the risks associated with the investment. Also, short selling implies borrowing funds from the broker and paying interest on this loan, which increases the costs. To eliminate the above aspects, short selling is eliminated from the optimization.

To utilize all of the assets included in the portfolio, additional constraint is enforced. Forcing the excel solver to have at least 1% of each asset included in the portfolio.

$$\forall w_i \geq 0,01 \quad (8)$$

This constraint provides more efficient diversification of the portfolio, increasing contribution of assets to overall portfolio metrics. The second important constraint is elimination of leverage.

$$\sum_i^N w_i = 1 \text{ for } i = [1; N] \quad (9)$$

As leverage increases the risk of not being able to return the funds borrowed and increases transaction costs; this constraint also holds the full employment of funds. Full employment makes efficient use of funds, investing all of the funds assigned for the investment without any residuals invested at a risk-free interest rate bank account.

Results and discussion

Table 1 provides bond yields for the past 12 years, except the S&P T-bills index 1-3 years, due to lack of information available. Bonds are taken with similar time to maturity where it is possible.

Data collected includes: two bonds with maturity of 3 years from Australia and Canada; as well as, a bond index of US T-bills with maturity of 1-3 years and European Central Bank bonds of all countries included in EU. Yield to maturity is used to provide information about all-possible gains that investor will receive throughout the holding period of the asset. From analysis of the data, it can be observed that, overall, in ECB, Australian and Canada bonds the YTM has been dropping down significantly. ECB debt securities have dropped its yield over 11.6 times during the 12-year period, even providing a return to its investors of only 0.08 % in 2016. There has been a similar trend in the Australian and Canadian bond yields, but only by 3 and 2.79 times respectively. During the same time frame, US T-bills index has been growing in its yields, from 1.78% to 2.71% in the past 9 years. Financial crisis in 2008 and 2014 has affected all of the countries analysed, and there is a significant trend that all of the bonds returns are growing back. Only the US market has recovered fully and shows growth. The evidence from the market clearly shows that the crisis in 2008 has significantly affected the whole economy, especially the government obligations of highly graded countries. The data clearly shows that the sub-prime mortgage crisis has affected the value of assets in investors' portfolios; credit crunch has lowered the liquidity of the debt securities, making the yields to drop down.

Bank deposits rates had a very similar dynamic as government obligations. Table 2 presents deposit interest rates for the Canadian market.

Interest rates on deposits in Canada show that the crisis in 2008 had a great impact on bank deposits, making interest rates to fall down from 2.08% in 2007 to 0.05% in three years. After a minor recovery in 2014, the second crisis for the analysed period has affected the world, making deposits to drop their return to 0.1% in 2015 and 2016. The correlation between the Canadian YTM of government obligations and its deposit interest rates shows a value of 0.9, making them almost perfectly correlated. The same situation can be observed with other countries included in the portfolio. Such a situation forces investors to search for alternatives with a better return but of the similar level of risks. According to Damodaran (2010) the only investment that could be counted as default risk-free are government obligations, due to a possibility of printing banknotes to fulfil its obligations, which makes government obligations a better alternative to bank deposits.

Table 3 contains the expected returns and standard deviations of each asset that are later included in the portfolio.

Geometric average is taken to calculate average return of the assets, which is used for portfolio return calculation. From the calculation, we can

observe that a Canadian 3-year bond has a lowest standard deviation for the analysed period; as well as, S&P index has a low standard deviation compared to ECB and Australian bonds. Australian debt instrument has a highest yield to maturity, providing its investors with the highest return out all of the chosen assets, but at the same time accepting the second highest standard deviation. Despite the fact that debt securities have been significantly dropping down in its yields, the volatility has dramatically increased during the past 12 years, due to several recessions. Central banks across the world have been keeping the interest rates at the historical lows for the past decade to boost up the spending in economy. This negatively affects the bond returns, but not only the coupon payments, but also the liquidity of the securities (Kiley & Roberts, 2017). Taking into account all of the negative aspects of highly rated government debt instruments, low risk investment that have not defaulted even during the severe recession. Pension funds and insurance companies are particularly searching for low risk investments to obtain a stable return. Nowadays, pension funds are highly regulated by the government authorities, enforcing the funds to invest only in government securities. Calculations show that the instruments are providing lower returns that in past periods; although, the performance is growing back with every year, providing a safe investment opportunity and attracting new funding.

Calculations of the expected return and standard deviation have been used to calculate the portfolio return with different weights. The modern portfolio theory has been used to calculate portfolio returns, and standard deviation of the portfolios with different allocations of asset. The results are provided in the Table 4.

W_i represents the part of total investment capital invested in a particular asset. Three main portfolios have been calculated: minimum variance, maximum expected return and maximum Sharpe ratio. The Excel solver has helped to optimize the portfolios with given constraints: full capital employment, no short sales and maximizing expected return for each level of risk. Also, a constraint of including, at least one percent of capital invested in each of the asset selected for optimization has been applied. The maximum Sharpe ratio for such a combination is 1.83, making this portfolio to be the most efficient. Compared to the general case of the US market, where 0.5 is a basis of low-risk long only portfolio at 2014, the international diversification has helped to increase the value; as also, the financial crisis of 2008 and 2014 have significantly affected standard deviations. The biggest problem is optimization of the ECB index due inefficiency of the assets, with lowest return and with second highest standard deviation. This shows that the EU government obligations on average provide a low ex-

pected return, but due to the sovereign debt crisis it bears a high-imbedded risk, which should be carefully evaluated and weighted before the investment. Even including assets with very low default risk (rated as AAA by the agencies), the overall portfolio standard deviation is high compared to the years before the subprime mortgage crisis. The point with a highest Sharpe ratio of calculated results is 1.83, causing the portfolio to bear extra 1.83% of return for each unit of standard deviation taken over risk-free asset. Compared to the Barclays aggregate bond index for the period from January 1993 to December 2012, where overall statistics shows that the return has been on average 6.34% with annualized standard deviation of 3.65%, this portfolio has a much lower standard deviation of 0.67%. At the same time, the portfolio has a much lower expected return of 2.05%.

The highest possible return for each level of risk is represented by the efficient frontier in Figure 1, where the tangency line represents the possibility for an investor to choose the inclusion of the risk free asset for diversification. The inclusion of risk-free asset reduces risk but at the same time lowers the expected return. Capital allocation line is tangent to the investment opportunity set at a point at which the Sharpe ratio is the highest for the portfolio.

Figure 1 represents graphically all possible combinations of assets, which maximizes expected return for each level of risk. Markowitz theory uses covariance between assets to represent the relationship, which helps to minimize the risk, at the same time keeping the expected return higher than it could be achieved by just investing in a single asset. Risk-free asset is chosen to be United Kingdom two-year gilts with the rate of 0.81%. The UK is one of the leading countries of the European Union, having its own currency which is independent of the others. While the UK is in the process of Brexit, it brings up the country power and makes it entirely independent of the EU decisions on the economic regulations of the EU.

The negative correlation between the assets contributes greatly to the diversification of the portfolio. As discussed previously, the negative correlation brings better diversification. If there has been a loss on one of the assets, due to the negative correlation, the second instrument would grow in its price and might cover up the loss. Table 5 presents the correlation matrix of the assets.

The selection has been done specifically from the AAA graded countries from different parts of the world. Correlation between the securities represents that all instruments are positively correlated between each other. Australia has a correlation of 0.6 with S&P index and S&P index has a correlation of 0.67 with ECB obligations. All of the other securities are highly correlated with each other with a value greater than 0.8, and in some cases

with a value of 0.97 and 0.92. The data shows that in the period of crisis and afterwards, the debt market across the world behaves in the same manner, making it difficult for the funds to diversify the portfolios. Despite the fact that the bonds have been particularly selected to provide a more efficient diversification, the highly graded bonds are commonly affected by the economic environment and most possess a strong positive correlation. Especially the Australian and ECB government obligations behave in positive directions to changes, if the yield to maturity of one assets drops down, the second one decreases by a similar magnitude. On the other hand, the correlation between Standard and Poors T-bills index has a close to zero coefficient, making the diversification to be more efficient, but the assets are not related in their changes

The risk-averse investors, such as low-risk hedge funds could also include a risk-free asset into the portfolio to minimise the volatility of the portfolio. Further additions of assets with different correlations could even minimise the volatility, even though increasing reward to volatility ratio for investors.

Comparing the simulated portfolio with the analysed bank deposits, we can summarise that investment in government securities provides a much better return. Putting a deposit in Euros into a German bank can bring annually 0.75% on average, contributing to a continuous compounding interest at the same maturity (3years) as bonds to be 1.24%, when simulated portfolio can bring 2.05%. Government obligations play a critical part in commercial banks activities to store funds, provide collateral and hold a liquid asset on the balance sheet. During the time of crisis, banks often try to increase their holdings of government debt as an interest rate pick up, to provide itself with a safe investment with higher yields than could be possibly obtained in the future (Gennaioli *et al.*, 2013). The current market situation with constant political fights with sanctions and Brexit does not allow investors to hope for a better future returns. Every day, the number of risk-averse investor grows, causing a demand for debt securities to increase. Hedge funds can provide international diversification with a small transaction costs and, at the same time, they provide a wide variety of investment alternatives for their clients. Low-risk portfolios are a good example of HF market activities that provide its clients a stable return. Pension funds and insurance companies are hunting for low-risk government obligations to be able to fulfil its daily activities.

Conclusions

The paper contributes to the field of the debt instruments portfolios, which has not been updated in the past years due to the growing popularity of more risky alternatives, which provide returns higher than the debt instruments, but the risks associated with such investments are tremendously higher. Mean-variance portfolio provides the most efficient investment allocation of the assets. The simulated portfolio provides the expected return of 2.05% and the standard deviation of 0.67%. International diversification helps to minimise unsystematic risk, at the same time boosting the expected return. Due to the changes in the market conditions, returns on the obligations have dropped significantly. However, it is still a low-risk investment with a stable return for the period of maturity. During the sovereign crisis, many countries have experienced difficulties with its debt obligations, but none of them have defaulted. Comparing with the US market low-risk portfolio Sharpe ratio in 2014, we can see that international diversification can increase the efficiency of investment. MPT and Sharpe ratio have contributed to this research tremendously, at the same time proving that the methods are valid even today.

The results show that the investing in government obligations portfolio can be a good alternative to a bank deposits, as it provides a stable return and, according to Damodaran (2012), fulfilling the conditions of risk-free investment. Simulation of a low risk hedge fund portfolio with debt instruments has shown that careful selection of highly graded government securities of different countries can allow investors to receive stable returns which are higher than deposits in countries at a similar level of development. High Sharpe ratio attracts institutional investors, such as pension funds and insurance companies to support its legal obligations and daily activities. The analysis provided the grounds for supporting low-risk hedge fund style portfolios, not only for institutional investors, but also for private ones. Most of the developed countries have a low deposit interest rates that could be outperformed by the debt portfolio. This research outlines advantages of international diversification, high-rated government obligations portfolio and low risk hedge funds activities.

In conclusion, this analysis confirms the stability of bond portfolios. During the period of instable financial markets, investors tend to stick to low risk stable alternatives, where a portfolio of government obligations is one of the preferred alternatives. Simulated portfolio has a low risk of default and volatility, as well as a constant coupon payment. Such portfolios are particularly appealing to pension funds, which are required by regulations to invest only in government obligations of highly rated countries.

Insurance companies are also interested in this type of investment, due to the necessity of holding liquid capital to fulfil its possible obligations to clients. Commercial banks often hold government obligations in its portfolios, due to lower default risk of government compared to the private or corporate instruments. During the crisis commercial banks often minimize their loans to private sector and focus on government debt to obtain high yields compared with future bond placements and to have a stable default-free income in the period of recession.

This model is a static representation of a portfolio, which considers a long position with expected return till the end of maturity of the assets. This model calculates the weights for the portfolio, which have to be constants and not adjusted throughout the investment horizon. In the real-case scenario, investors always tend to adjust the portfolios with respect to the changes in the investing environment, changing IR and possible. The simulated portfolio does not take into account transaction costs, currencies exchange rates and risks; this is a future plan for development of this analysis. Future modifications also include active management of portfolio; this will contribute to higher possible returns and reduced risks of unexpected events. ECB index (that have been included in portfolio) appeared to be inefficient as it has one of the highest volatilities with the lowest returns, due to that excel solver has tried to minimise its asset weight to the lowest possible constraint limit.

Reference

- Agarwal, V., & Naik, N. (2003). Risks and portfolio decisions involving hedge funds. *Review of Financial Studies*, 17(1). doi: 10.1093/rfs/hhg044.
- Amenc, N., & Martellini, L. (2002). Portfolio optimization and hedge fund style allocation decisions. *SSRN Electronic Journal*. doi: 10.2139/ssrn.305006.
- Benczur, P. (2002). Identifying sovereign bond risks. *SSRN Electronic Journal*. doi: 10.2139/ssrn.296237.
- Bernoth, K., von Hagen, J., & Schuknecht, L. (2012). Sovereign risk premiums in the European government bond market. *Journal of International Money and Finance*, 31(5). doi: 10.1016/j.jimonfin.2011.12.006.
- Bodie, Z., Kane, A., & Marcus, A. (2013). *Essentials of investments*. New York: McGraw-Hill/Irwin.
- Brennan, M. (2018). Capital asset pricing model. In *The new Palgrave dictionary of economics*. Palgrave Macmillan.
- Brooks, C., & Kat, H. (2002). The statistical properties of hedge fund index returns and their implications for investors. *Journal of Alternative Investments*, 5(2). doi: 10.3905/jai.2002.319053.

- Caks, J. (1977). The coupon effect on yield to maturity. *Journal of Finance*, 32(1). doi: 10.2307/2326906.
- Canada – Economic Indicators (2018). Retrieved from <https://tradingeconomics.com/canada/indicators>.
- Chow, T., Hsu, J., Kuo, L., & Li, F. (2014). A study of low-volatility portfolio construction methods. *Journal of Portfolio Management*, 40(4). doi: 10.3905/jpm.2014.40.4.089.
- Claessens, S., Klingebiel, D., & Schmukler, S. (2007). Government bonds in domestic and foreign currency: the role of institutional and macroeconomic factors. *Review of International Economics*, 15(2). doi: 10.1111/j.1467-9396.2007.00682.x.
- Damodaran, A. (2010). Into the abyss: what if nothing is risk free? *SSRN Electronic Journal*. doi: 10.2139/ssrn.1648164.
- De Santis, G., & Gerard, B. (1998). How big is the premium for currency risk? *Journal of Financial Economics*, 3(49).
- Eling, M., & Schuhmacher, F. (2007). Does the choice of performance measure influence the evaluation of hedge funds? *Journal of Banking & Finance*, 31(9). doi: 10.1016/j.jbankfin.2006.09.015.
- Fraś, A. (2018). The relation between management fees and the mutual funds' performance in Poland in 2015. *Oeconomia Copernicana*, 9(2). doi: 10.24136/10.24136/oc.2018.013.
- French, K., Schwert, G., & Stambaugh, R. (1987). Expected stock returns and volatility. *Journal of Financial Economics*, 19(1). doi: 10.1016/0304-405x(87)90026-2.
- Gennaioli, N., Martin, A., & Rossi, S. (2013). Banks, government bonds, and default: what do the data say? *SSRN Electronic Journal*. doi: 10.2139/ssrn.2308340.
- Giamouridis, D., & Vrontos, I. (2005). Hedge fund portfolio construction: a comparison of static and dynamic approaches. *SSRN Electronic Journal*. doi: 10.2139/ssrn.682384.
- Hawawini, G. (1984). On the relationship between Macaulay's bond duration and the term to maturity. *Economics Letters*, 16(3-4). doi: 10.1016/0165-1765(84)90185-x.
- Hennessee Fixed Income Index (2018). Retrieved from <http://www.hennesseeigroup.com/indices/returns/strategy/fixedincome.html>.
- Hopewell, M., & Kaufman, G. (2018). Bond price volatility and term to maturity: a generalized respecification. In G. Hawawini (Ed.). *Bond Duration and Immunization*. Taylor Francis.
- Hosseini, A., Wang, J., & Hosseini, S. (2013). A recurrent neural network for solving a class of generalized convex optimization problems. *Neural Networks*, 44. doi: 10.1016/j.neunet.2013.03.010.
- Kiley, M., & Roberts, J. (2017). Monetary policy in a low interest rate world. *Brookings Papers on Economic Activity*, 1. doi: 10.1353/eca.2017.0004.
- Lamm, R. (2003). Asymmetric returns and optimal hedge fund portfolios. *Journal of Alternative Investments*, 6(2). doi: 10.3905/jai.2003.319088.

- Lhabitant, F. (2001). Assessing market risk for hedge funds and hedge funds portfolios. *SSRN Electronic Journal*. doi: 10.2139/ssrn.268527.
- Lim, A., & Zhou, X. (2002). Mean-variance portfolio selection with random parameters in a complete market. *Mathematics of Operations Research*, 27(1). doi: 10.1287/moor.27.1.101.337.
- Lo, A., & MacKinlay, A. (1988). Stock market prices do not follow random walks: evidence from a simple specification test. *Review of Financial Studies*, 1(1). doi: 10.1093/rfs/1.1.41.
- Machina, M., & Rothschild, M. (2008). Risk. In *The new Palgrave dictionary of economics*. Palgrave Macmillan.
- Markowitz, H. (1952). Portfolio selection. *Journal of Finance*, 7(1). doi: 10.2307/2975974.
- Merton, R. (1974). On the pricing of corporate debt: the risk structure of interest rates. *Journal of Finance*, 29(2). doi: 10.1111/j.1540-6261.1974.tb03058.x.
- Meluzín, T., Balcerzak, A.P., Pietrzak, M. B., Zinecker, M., & Doubravský, K. (2018a). The impact of rumours related to political and macroeconomic uncertainty on IPO success: evidence from a qualitative model. *Transformations in Business & Economics*, 17(2)(44).
- Meluzín, T., Zinecker, M., Balcerzak, A.P., Doubravský, K., Pietrzak, M. B., & Dohnal, M. (2018b). The timing of initial public offerings – non-numerical model based on qualitative trends. *Journal of Business Economics and Management*, 19(1).doi: 10.3846/jbem.2018.1539.
- Meluzín, T., Pietrzak, M. B., Balcerzak, A. P., Zinecker, M., Doubravský, K., & Dohnal, M. (2017). Rumours related to political instability and their impact on IPOs: the use of qualitative modeling with incomplete knowledge. *Polish Journal of Management Studies*, 16(2). doi: 10.17512/pjms.2017.16.2.15.
- Pogue, G. (1970). An extension of the Markowitz portfolio selection model to include variable transactions' costs, short sales, leverage policies and taxes. *Journal of Finance*, 25(5). doi: 10.2307/2325576.
- Szumilo, N., Bienert, S., Łaskiewicz, E., Pietrzak, M. B. & Balcerzak, A. P. (2018). The real alternative? A comparison of German real estate returns with bonds and stocks. *Journal of Property Investment & Finance*, 36(1). doi: 10.1108/JPIF-02-2017-0012.
- S&P U.S. Treasury TIPS 1-3 Year Index - S&P Dow Jones Indices. (2018). Retrieved from <https://us.spindices.com/indices/fixed-income/sp-us-treasury-tips-1-3-year-index>.
- Sharpe, W. (1966). Mutual fund performance. *Journal of Business*, 39(S1). doi: 10.1086/294846.
- Vukovic, D. B., Hanic, E., & Hanic, H. (2017). Financial integration in the European Union — the impact of the crisis on the bond market. *Equilibrium. Quarterly Journal of Economics and Economic Policy*, 12(2). doi: 10.24136/eq.v12i1.10.
- Vyklyuk, Y., Vukovic, D., & Jovanovic, A (2013). Forex prediction with neural network: USD/EUR currency pair. *Actual Problem of Economics*, 10(148).

World Government Bonds – Investing.com. (2018). Retrieved from <https://www.investing.com/rates-bonds/world-government-bonds>.

World Savings Account Rates by Country- Compare Savings Accounts around the world. (2018). Retrieved from <https://www.deposits.org/savings-accounts.html>.

Annex

Table 1. Bond yields

%	ECB all	S&P T-bills1-3y index	Australia 3year	Canada 3 year
2018	0.37	2.71	2.07	1.65
2017	0.15	1.75	1.74	0.71
2016	0.08	1.18	1.88	0.54
2015	0.49	1.14	2.27	0.82
2014	1.13	1.03	2.88	1.24
2013	0.57	0.71	2.62	1.15
2012	1.13	0.77	3.47	1.23
2011	2.65	1.42	4.86	1.81
2010	1.94	1.78	4.84	1.91
2009	2.73	2.4	4.09	2.02
2008	4.28	4.39	6.71	3.38
2007	4.30		6.38	4.60
Expected return	0.92	1.28	3.30	1.47
Std	1.53	0.62	1.72	1.17

Source: authors based on S&P (2018).

Table 2. Interest rates on deposits on Canadian market

Year	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
Deposit interest per year (%)	2.08	1.5	0.1	0.05	0.47	0.48	0.55	0.55	0.1	0.1

Source: Tradingeconomics.com (2018).

Table 3. Expected returns and standard deviations

Expected return (%)	0.92	1.51	3.30	1.47
Std (%)	1.53	1.08	1.72	1.17

Table 4. Portfolio returns with different weighs

w1	w2	w3	w4	Summ (wi)	E(Rp) (%)	Std port (%)	Sharpe	
0.01	0.01	0.97	0.01	1	3.24	1.70	1.43	Max expected
0.01	0.14	0.84	0.01	1	3.00	1.47	1.49	
0.01	0.32	0.66	0.01	1	2.68	1.16	1.61	
0.01	0.68	0.30	0.01	1	2.05	0.67	1.83	Max Sharpe
0.01	0.77	0.21	0.01	1	1.87	0.60	1.77	
0.13	0.85	0.01	0.01	1	1.45	0.57	1.13	Min variance
1.00	0.00	0.00	0.00	1	0.92	1.53	0.08	Min Sharpe

Table 5. Correlation matrix of the assets

Correlation	ECB all	S&P T-biils1-3y index	Australia 3year	Canada 3 year
ECB all	1.00			
S&P T-biils1-3y index	0.67	1.00		
Australia 3year	0.97	0.60	1.00	
Canada 3 year	0.92	0.83	0.89	1.00

Figure 1. Efficiency frontier