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DESCRIPTION OF SELECTED PNEUMATIC ELEMENTS AND SYSTEMS USING FRACTIONAL CALCULUS

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Abstract – The paper presents the application of fractional calculus to describe the dynamics of selected pneumatic elements and systems. In the construction of mathematical models of the analysed dynamic systems, the Riemann-Liouville definition of differintegral of non- integer order was used. For the analysed model, transfer function of integer and non-integer order was determined. Functions describing characteristics in frequency domains were determined, whereas the characteristics of the elements and systems were obtained by means of computer simulation. MATLAB programme were used for the simulation research.

Key words – fractional calculus, pressure transducer.

INTRODUCTION

The paper presents the mathematical analysis of the pressure chamber with inlet pipe and pneumatic cascade [5], [6], [7], [8], [9], [10] and [11] described with the differential calculus of non-integer orders (fractional calculus) [1], [2], [3] and [14].

Differential equations of integer and non-integer order were introduced and became the basis for deriving equations describing time characteristics (pulse and step) and frequency characteristics (logarithmic amplitude and phase characteristics) for each tested pneumatic system. Simulations of derived equations was performed using MATLAB&Simulink software [10, 11] and [13], obtaining frequency characteristics of the tested systems for integer and non-integer orders.

I. MEMBRANE PRESSURE TRANSDUCER

Simulation tests of the membrane pressure transducer were performed using a classical and fractional differential calculus [8], [9], [10] and [11]. The tested transducer was made of a pressure chamber and an inlet pipe, which supplied a working medium (air). To determine how the connection of the intake pipe to the transducer chamber affected its dynamic properties, the acoustic system shown schematically in figure 1 is considered.

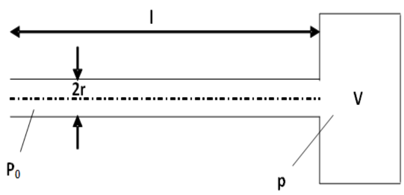

Fig. 1. **Pressure chamber with inlet pipe: r, l - tube dimensions, p_0 - input pressure (force), p - pressure in transducer chamber**

The relationship that binds the output signal p(t) (pressure inside the chamber) to the signal $p_0(t)$ (pressure at the open end) can be represented as:

$$\frac{d^2 p(t)}{dt^2} + \frac{R_p}{L_p}\frac{dp(t)}{dt} + \frac{1}{L_p C_p} p(t) = \frac{1}{L_p C_p} p_0(t) \tag{1}$$

Constants occurring in equation (1) can be represented as:

$$C_p = \frac{V}{\rho c^2} \quad \left[Ns^2 m^{-5}\right]$$

$$L_p = \frac{4l\rho}{3\pi r^2} \quad \left[m^5 N^{-1}\right] \qquad \text{(2a, b, c)}$$

$$R_p = \frac{8\eta l}{\pi r^2} \quad \left[Nsm^{-5}\right]$$

where:

p[kgm^{-3}] - density of gas;
η [$kgm^{-1}s^{-1}$] - dynamic viscosity;
Cp [Ns^2m^{-5}] - pneumatic capacity (gas compressibility);
p (t) [Pa] - pressure in transducer chamber;
p_0 (t) [Pa] - input pressure;
V [m^3] - transducer chamber volume;
Lp [m^3N^{-1}] - pneumatic induction (gas inertia);
Rp [Nsm^{-5}] - flow resistance;
c [ms^{-1}] - speed of sound in the gas;
r, l [m] - dimensions of the inlet pipe.

By specifying the pulsatance ω_0 and damping ratio ξ as:

$$\omega_0 = \frac{1}{\sqrt{L_p C_p}} = \sqrt{\frac{3\pi^2 c^2}{4lV}}$$

$$\xi = \frac{R_p C_p \omega_0}{2} = \qquad \text{(3a, 3b)}$$

$$= \frac{R_p}{2}\sqrt{\frac{C_p}{L_p}} = 2\frac{\eta\sqrt{\frac{3lV}{\pi}}}{r\rho c} = 2\sqrt{\frac{3\eta^2 lV}{\pi r^2 \rho^2 c^2}}$$

equation (1) finally assumes the form:

$$\frac{d^2 p(t)}{dt^2} + 2\xi\omega_0 \frac{dp(t)}{dt} + \omega_0^2 p(t) = \omega_0^2 p_0(t) \qquad (4)$$

Equation (4) is a mathematical description of the dynamics of the analyzed pneumatic system, using classical differential calculus (of integer orders).The impulse response of the analyzed pneumatic system is given by:

$$g(t) = \frac{\omega_0}{\sqrt{1-\xi^2}} e^{-\xi\omega_0 t} \sin \omega_0 \sqrt{1-\xi^2}\, t \qquad (5)$$

The step response of the system is expressed by:

$$h(t) = 1 - \frac{e^{-\omega_0 \xi t}}{\sqrt{1-\xi^2}} \sin\left(\omega_0 \sqrt{1-\xi^2}\, t + \varphi\right) \qquad (6)$$

where:

$$\varphi = arctg \frac{\sqrt{1-\xi^2}}{\xi} \qquad (7)$$

Given that the derivatives of integer orders in the fractional calculus are a special case of fractional derivatives, equation (4) can be written as:

$$^{RL}_{0}D_t^{2\nu} p(t) + 2\xi\omega_0 \, ^{RL}_{0}D_t^{\nu} p(t) + \omega_0^2 p(t) = \omega_0^2 p_0(t) \qquad (8)$$

where: $\nu > 0$.

In order to determine the pressure in the transducer chamber, the definition of Riemann - Liouville differintegral of non-integer order was used, defined by a following formula:

$$^{RL}_{a}D_t^{\alpha} f(t) = \frac{1}{\Gamma(k-\alpha)} \frac{d^k}{dt^k} \int_a^t (t-\tau)^{k-\alpha-1} f(\tau)\, d\tau \qquad (9)$$

where:

α – the order of integration within bounds (a, t) of the function $f(t)$, $k-1 \le a \le k$ and: $\alpha \in R^+$, $\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt$ - Euler's gamma function

The Laplace transform for a fractional derivative defined by Riemann - Liouville takes the form:

$$L\left[^{R-L}_{0}D_t^{\alpha} f(t)\right] = s^\alpha F(s) - \sum_{k=0}^{j-1} s^k \, ^{R-L}_{0}D_t^{\alpha-k-1} f(0) \qquad (10)$$

where: $j-1 \le \alpha \le j \in N$

The practical application of the formula determining the Laplace transform of a Riemann-Liouville fractional derivative faces some difficulties related to the lack of physical interpretation of the initial values of successive derivatives of fractional orders. Assuming zero initial conditions, the difficulties associated with their physical interpretation will be eliminated.

Using the Laplace transform to equation (8), for zero initial conditions, we obtain:

$$s^{2\nu} p(s) + 2\xi\omega_0 s^\nu p(s) + \omega_0^2 p(s) = \omega_0^2 p_0(s) \qquad (11)$$

Thus the transfer function of non-integer order of the analyzed pneumatic system is obtained:

$$G^{(\nu)}(s) = \frac{\omega_0^2}{s^{2\nu} + 2\xi\omega_0 s^\nu + \omega_0^2} \qquad (12)$$

Transfer function denominator of non-integer order has two complex roots as the system damping is $\xi < 1$.

II. FREQUENCY RESPONSE OF THE MEMBRANE PRESSURE TRANSDUCER

In order to determine the relationships describing the frequency response, the spectral transfer function of the tested transducer was determined. Substituting:

$$s = j\omega = \omega e^{j\frac{\pi}{2}} = \omega\left[\cos\left(\frac{\pi}{2}\right) + j\sin\left(\frac{\pi}{2}\right)\right] \quad (13)$$

to the formula (12), the spectral transfer function of the transducer is obtained:

$$G^{(\nu)}(j\omega) = \frac{\omega_0^{\,2}}{(j\omega)^{2\nu} + 2\xi\omega_0 (j\omega)^{\nu} + \omega_0^{\,2}} \quad (14a)$$

$$G^{(\nu)}(j\omega) = \frac{\omega_0^{\,2}}{\omega^{2\nu}\left[\cos(\nu\pi) + j\sin(\nu\pi)\right] + 2\xi\omega_0\omega^{\nu}\left[\cos\left(\nu\frac{\pi}{2}\right) + j\sin\left(\nu\frac{\pi}{2}\right)\right] + \omega_0^{\,2}} \quad (14b)$$

By making elementary transformations, the real and imaginary part of the spectral transfer function was calculated:

$$G^{(\nu)}(j\omega) = P^{(\nu)}(\omega) + jQ^{(\nu)}(\omega) \quad (15)$$

where:

$$P^{(\nu)}(\omega) = \frac{\omega_0^{\,2}\omega^{2\nu}\cos(\nu\pi) + 2\xi\omega_0^{\,3}\omega^{\nu}\cos\left(\frac{\nu\pi}{2}\right) + \omega_0^{\,4}}{\left[\omega^{2\nu}\cos(\nu\pi) + 2\xi\omega_0\omega^{\nu}\cos\left(\frac{\nu\pi}{2}\right) + \omega_0^{\,2}\right]^2 + \left[\omega^{2\nu}\sin(\nu\pi) + 2\xi\omega^{\nu}\omega_0\sin\left(\frac{\nu\pi}{2}\right)\right]^2} \quad (16a)$$

$$Q^{(\nu)}(\omega) = -\frac{\omega_0^{\,2}\omega^{2\nu}\sin(\nu\pi) + 2\xi\omega_0^{\,3}\omega^{\nu}\sin\left(\frac{\nu\pi}{2}\right)}{\left[\omega^{2\nu}\cos(\nu\pi) + 2\xi\omega_0\omega^{\nu}\cos\left(\frac{\nu\pi}{2}\right) + \omega_0^{\,2}\right]^2 + \left[\omega^{2\nu}\sin(\nu\pi) + 2\xi\omega_0\omega^{\nu}\sin\left(\frac{\nu\pi}{2}\right)\right]^2} \quad (16b)$$

Knowing the real and imaginary part of the spectral transfer function of the transducer, one can determine the equation describing the logarithmic amplitude step:

$$L^{(\nu)}(\omega) = 20\log\sqrt{\left[P^{(\nu)}(\omega)\right]^2 + \left[Q^{(\nu)}(\omega)\right]^2} \quad (17)$$

and the equation describing the logarithmic phase step:

$$\varphi^{(\nu)}(\omega) = arctg\left[\frac{Q^{(\nu)}(\omega)}{P^{(\nu)}(\omega)}\right] = -arctg\left[\frac{\omega^{2\nu}\sin(\nu\pi) + 2\xi\omega_0\omega^{\nu}\sin\left(\frac{\nu\pi}{2}\right)}{\omega^{2\nu}\cos(\nu\pi) + 2\xi\omega_0\omega^{\nu}\cos\left(\frac{\nu\pi}{2}\right) + \omega_0^{\,2}}\right] \quad (18)$$

In order to verify the relationships describing logarithmic steps of amplitude (17) and phase (18) of the tested transducer, the pneumatic pressure transducer was modeled in the MATLAB environment, described by means of ordinary differential equation and differential equation with derivatives of non-integer order. Describing the transducer with the use of a differential equation of non-integer orders, the parameter $\nu = 1$ was assumed and the obtained logarithmic amplitude and phase steps were compared to the logarithmic amplitude and phase steps obtained from the transducer description by means of the ordinary differential equation.

The simulations assumed:

- pulsatance $\omega_0 = 500\,[rad\,/\,s]$,
- damping ratio $\xi = 0{,}2$

The transfer function of the pneumatic pressure transducer, calculated from the ordinary differential equation, has the form:

$$G(s) = \frac{p(s)}{p_0(s)} = \frac{\omega_0^{\,2}}{s^2 + 2\xi\omega_0 s + \omega_0^{\,2}} \quad (19)$$

By performing the simulation of equation (19) which presents the dynamics of the phenomena occurring in the analyzed pneumatic system, in the MATLAB programming environment, the frequency response presented in figure 2 was obtained:

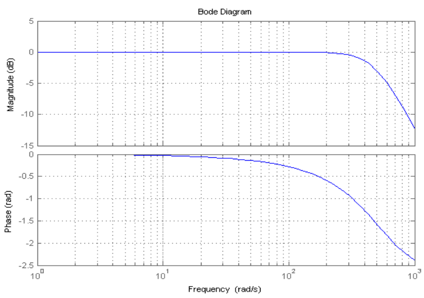

Fig. 2. **Logarithmic frequency response of the transducer described by the ordinary differential equation**

When simulating equations (17) and (18) in the MATLAB environment which describe a pneumatic pressure transmitter by means of a differential equation of non-integer order, assuming a coefficient $\nu = 1$ for damping $\xi = 0{,}8$, the response shown in figure 3 was obtained:

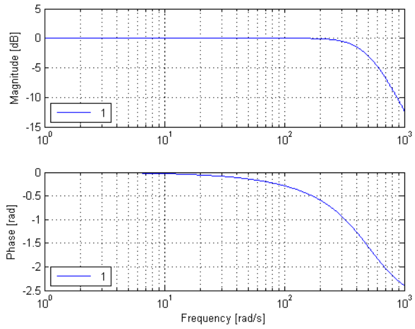

Fig. 3. **Logarithmic frequency response of a pneumatic transducer described by means of a differential equation with non-integer order for $\nu = 1$ (equation 17 and 18)**

Logarithmic frequency response of amplitude and phase presented by the simulation of ordinary differential equation (figure 2), coincide with frequency response obtained by the simulation of the equations describing logarithmic response of amplitude (17) and phase (18), obtained from the equation of the transducer described with the help of non-integer orders (figure 5) for the parameter $\nu = 1$.

In order to obtain a Bode plot, the equations (17) and (18) were simulated by writing an appropriate program in the MATLAB environment. Written in the MATLAB environment, the program allows analyzing the transducer for different orders of derivatives, with any step, because the order was given as a parameter. The simulation results for the selected values of parameter *v* are shown in figure 4 and figure 5.

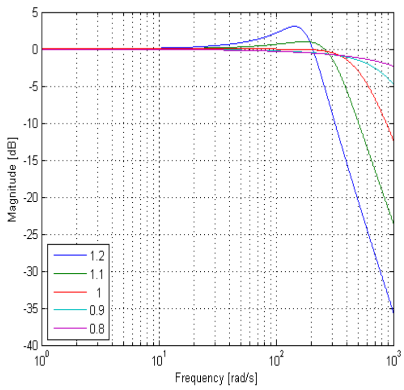

Fig. 4. **Logarithmic amplitude response of a pneumatic transducer described by means of differential equation with fractional derivatives of non-integer orders in the range (0.8-1.2)**

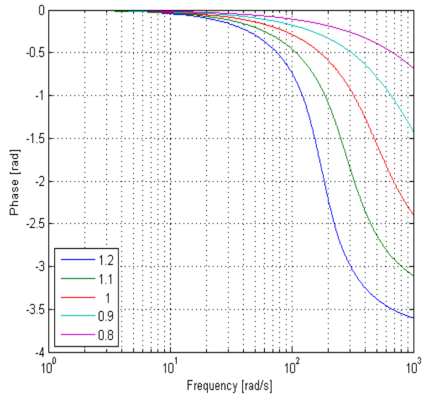

Fig. 5. **Logarithmic phase response of a pneumatic transducer described by means of differential equation with fractional derivatives of non-integer orders in the range (0.8-1.2)**

The analysis of the responses shows that for the parameter $\nu < 1$, the logarithmic amplitude responses (figure 4) are monotonically decreasing functions. For the parameter $\nu > 1$, the logarithmic amplitude responses have a maximum depending on the order of the differential. The maximum is achieved with resonant frequency $\omega_0 = 110\,\frac{rad}{s}$.

Increasing the order of derivative, the frequency responses acquire the character of a second-order oscillatory element, and while decreasing the order of the derivative, the responses acquire the character of the first order inertial element.

Decreasing the order of the derivative causes the transducer to become more linear, which allows the scope of work to be increased.

Increasing the parameter ν above one results in resonance, although it should not be visible in the response, because the simulation was carried out for the damping $\xi = 0{,}8$. The model then does not reflect the actual layout.

III. MATHEMATICAL MODEL OF A PNEUMATIC CASCADE DESCRIBED WITH A FRACTIONAL CALCULUS

Fig.6 shows the diagram of the analyzed pneumatic cascade:

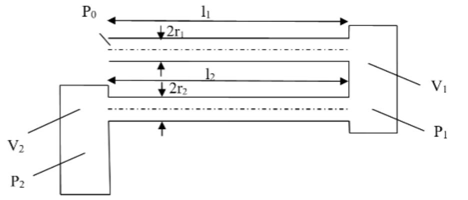

Fig. 6. **Diagram of a two-chamber pneumatic cascade**

Fig. 7 shows a block diagram of a two-chamber pneumatic cascade:

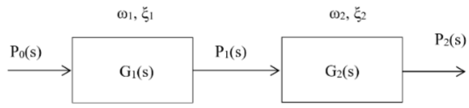

Fig. 7. **Block diagram of a two-chamber pneumatic cascade**

Assuming the linearity of the model, the equation describing the dynamics of the diaphragm pressure transmitter can be written in the form of a system of differential equations:

$$\frac{d^2 p_1(t)}{dt^2} + 2\xi_1\omega_1 \frac{dp_1(t)}{dt} + \omega_1^2 p_1(t) = \omega_1^2 p_0(t) \tag{20a}$$

$$\frac{d^2 p_2(t)}{dt^2} + 2\xi_2\omega_2 \frac{dp_2(t)}{dt} + \omega_2^2 p(t) = \omega_2^2 p_1(t) \tag{20b}$$

where:

$\omega_{1,2}\left(rad/s\right)$ - the pulsatance of another elementary pneumatic system,

$\xi_{1,2} < 1$ - the damping ratio of another elementary pneumatic system included in the pneumatic cascade.

$$\omega_{1,2} = \frac{1}{\sqrt{L_{p1,p2}C_{p1,p2}}} = \sqrt{\frac{3\pi r_{1,2}^2 c^2}{4 l_{1,2} V_{1,2}}} \tag{21a}$$

$$\zeta_{1,2} = \frac{L_{p1,p2}C_{p1,p2}\omega_{1,2}}{2} = 2\frac{\eta\sqrt{\frac{3l_{1,2}V_{1,2}}{\pi}}}{r_{1,2}\rho c} = 2\sqrt{\frac{3\eta^2 l_{1,2}V_{1,2}}{\pi r_{1,2}^2 \rho^2 c^2}} \tag{22b}$$

where in:

$C_p\left[Ns^2m^{-5}\right]$ - pneumatic capacity in another element of the pneumatic system;

$L_p\left[m^3N^{-1}\right]$ - pneumatic induction in another element of the pneumatic system;

$R_p\left[Nsm^{-5}\right]$ - flow resistance in another element of the pneumatic system;

$V\left[m^3\right]$ - volume of another transducer chamber,

$c\left[m/s\right]$ - speed of sound in the gas filling the system;

$l\left[m\right]$ - length of another inlet pipe;

$r\left[m\right]$ - radius of another inlet tube;

$\rho\left[kgm^{-3}\right]$ - gas density;

$\eta\left[kgm^{-1}s^{-1}\right]$ - dynamic viscosity.

Equations (20a,b) written with the help of fractional calculus take the following form:

$$\begin{aligned} {}_0^{RL}D_t^{2v} p_1(t) + 2\xi_1\omega_1\, {}_0^{RL}D_t^{v} p_1(t) + \omega_1^2 p_1(t) = \omega_1^2 p_0(t) \\ {}_0^{RL}D_t^{2v} p_2(t) + 2\xi_2\omega_2\, {}_0^{RL}D_t^{v} p_2(t) + \omega_2^2 p_2(t) = \omega_2^2 p_1(t) \end{aligned} \tag{22}$$

where: $v > 0$.

Using the Laplace transform to equation (22), for zero initial conditions, we obtain:

$$\begin{aligned} \left(s^{2v} + 2\xi_1\omega_1 s^v + \omega_1^2\right) p_1(s) = \omega_1^2 p_0(s) \\ \left(s^{2v} + 2\xi_2\omega_2 s^v + \omega_2^2\right) p_2(s) = \omega_2^2 p_1(s) \end{aligned} \tag{23}$$

Thus the transfer function of non-integer order of the analyzed pneumatic system is obtained:

$$\begin{aligned} G_1^{(v)}(s) = \frac{p_1(s)}{p_0(s)} = \frac{\omega_1^2}{s^{2v} + 2\xi_1\omega_1 s^v + \omega_1^2} \\ G_2^{(v)}(s) = \frac{p_2(s)}{p_1(s)} = \frac{\omega_2^2}{s^{2v} + 2\xi_2\omega_2 s^v + \omega_2^2} \end{aligned} \tag{24}$$

The transfer function of the analyzed system takes the form:

$$G^{(v)}(s)=\frac{p_2(s)}{p_0(s)}=G_1^{(v)}(s)G_2^{(v)}(s)$$

$$G^{(v)}(s)=\frac{\omega_1^2\omega_2^2}{s^{4v}+(2\xi_1\omega_1+2\xi_2\omega_2)s^{3v}+(\omega_1^2+4\xi_1\xi_2\omega_1\omega_2+\omega_2^2)s^{2v}+(2\xi_1\omega_1\omega_2^2+2\xi_2\omega_1^2\omega_2)s^v+\omega_1^2\omega_2^2} \quad (25)$$

For the formula (25), we obtain the spectral transfer function of the tested transducer:

$$G^v(j\omega)=\frac{\omega_1^2\omega_2^2}{\omega^{4v}\left[\cos(2\pi v)+j\sin(2\pi v)\right]+(2\xi_1\omega_1+2\xi_2\omega_2)\omega^{3v}\left[\cos\left(\frac{3\pi v}{2}\right)+j\sin\left(\frac{3\pi v}{2}\right)\right]+(\omega_1^2+4\xi_1\xi_2\omega_1\omega_2+\omega_2^2)\omega^{2v}\left[\cos(\pi v)+j\sin(\pi v)\right]+(2\xi_1\omega_1\omega_2^2+2\xi_2\omega_1^2\omega_2)\omega^v\left[\cos\left(\frac{\pi v}{2}\right)+j\sin\left(\frac{\pi v}{2}\right)\right]+\omega_1^2\omega_2^2} \quad (26)$$

By making elementary transformations, the real and imaginary part of the spectral transfer function is calculated:

$$G^{(v)}(j\omega)=P^{(v)}(\omega)+jQ^{(v)}(\omega) \quad (27)$$

where:

Knowing the real and imaginary part of the spectral transfer function of the transducer, one can determine the equation describing the logarithmic amplitude step:

$$L^{(v)}(\omega)=20log\sqrt{\left[P^{(v)}(\omega)\right]^2+\left[Q^{(v)}(\omega)\right]^2} \quad (28)$$

Using the program written in the MATLAB environment, which was used for conducting the simulations of the equations describing the Bode plot of the membrane pressure transducer, a response in the form of plots of logarithmic magnitude and phase of the analyzed pneumatic cascade was obtained. The plots are presented in fig. 8 and fig. 9.

The determined frequency response (fig. 8 and fig. 9) correctly reflects the dynamics of the model. For the parameter $v=1$, the logarithmic amplitude response (fig. 8) and phase response (fig. 9) coincide with the known responses of the 4th order oscillation units. From the amplitude response (fig. 8), one can read the gain decrease, which is $-80dB/dek$, and from the phase response (fig. 9), the phase shift $\varphi=-2\pi$ for the parameter $v=1$, as it is in the classic oscillation section of the 4th order.

The analysis of frequency responses (fig. 8 and fig. 9) shows that the resonant pulsation depends on the parameter v, and hence on the order of the differential, in the differential equation describing the studied system. By reducing the order, the resonant pulsation increases. Hence, the smaller the phase shift of the system is, the smaller the order of the differential.

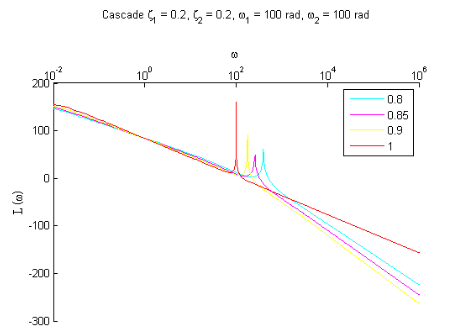

Fig. 8. **Logarithmic amplitude response of a pneumatic cascade described by means of differential equation with fractional derivatives of non-integer orders for the parameter v in the range (0.8-1.2) [authors' own elaboration]**

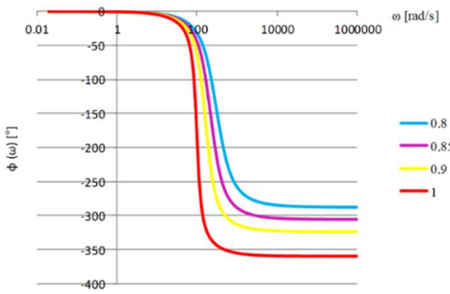

Fig. 9. **Logarithmic phase response of a pneumatic cascade described by means of differential equation with fractional derivatives of non-integer orders for the parameter v in the range (0.8-1.2) [authors' own elaboration]**

IV. Conclusions

The obtained responses, which arose from the simulation of the dependencies resulting from the solution of differential equations of integer orders, overlap the responses of non-integer orders obtained from the solution of differential equations of non-integer order for the parameter $v=1$. This is confirmed by the fact that the classical differential calculus is a special case of the differential calculus of any arbitrary order, and thus it proves that mathematical models have been properly developed.

The use of the description of dynamic properties of pneumatic systems based on the fractional calculus will allow to analyze the properties of a wide class of pneumatic systems of arbitrary orders in the future. Fractional calculus is particularly useful in building dynamic models of mathematical systems working in conditions that cannot be described with differential equations of integer orders. This can be deduced by analyzing systems such as the long electric line of infinitely large length or the supercapacitor of a few thousand Farads, which are now also described with fractional calculus.

WYBRANE ELEMENTY I UKŁADY PNEUMATYCZNE OPISANE ZA POMOCĄ RACHUNKU RÓŻNICZKOWEGO NIECAŁKOWITYCH RZĘDÓW

W artykule przedstawiono zastosowanie rachunku różniczkowego niecałkowitych rzędów (ang. fractional calculus) do opisu dynamiki zjawisk układów pneumatycznych wybranych elementów i układów. W budowie modeli matematycznych, analizowanych układów dynamicznych, wykorzystano definicję Riemanna–Liouville'a pochodno–całki niecałkowitego rzędu. Dla analizowanego modelu, wyznaczono transmitancję operatorową całkowitego i niecałkowitego rzędu. Wyznaczono zależności opisujące charakterystyki częstotliwościowe, na drodze symulacji komputerowej uzyskano charakterystyki analizowanych układów. Do badań symulacyjnych wykorzystano oprogramowanie MATLAB.

Słowa kluczowe: rachunek różniczkowy niecałkowitych rzędów

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