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Intra-industry trade in differentiated and homogenous commodities: Brander and Krugman models unified

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Abstract

Research background: This paper extends the early papers by Brander (1981) and Brander and Krugman (1983) who used a simple partial equilibrium Cournot duopoly to a full general equilibrium setting. The explanations of intra-industry trade can be based either on oligopolistic reciprocal dumping idea (Brander, 1981) or product differentiation (Dixit and Norman, 1980; Krugman, 1979, 1980, 1981; Lancaster, 1980; Helpman, 1981). In this paper we combine both explanations in a unified general equilibrium model.

Purpose of the article: We develop a two-sector, one-factor general equilibrium model, in which the first sector produces a differentiated good under monopolistic competition and the second sector produces a homogenous good under Cournot oligopolistic competition. In this paper, we study how competition between domestic and foreign firms resulting from trade liberalization affects intra-industry trade in both sectors.

Methods: The paper develops a two-sector model based on several assumptions. Consumers have a two-tier utility function of the Cobb-Douglas-Spence-Dixit-Stiglitz form. Firms operate in two sectors and produce goods under increasing returns to scale resulting from the existence of fixed costs. One sector produces homogenous good under Cournot competition, and the second one produces a differentiated product in under Chamberlinian monopolistic

competition. Free entry is assumed in both sectors. Labor is assumed to be the only factor of production with perfect mobility and full employment.

Findings & Value added: In contrast to previous papers, our study is based on a full general equilibrium Cournot oligopoly framework with many firms. Moreover, we endogenize the number of firms and study the resulting trading equilibria. Therefore, this paper can be regarded as the extension and unification of the early papers by Brander (1981), Brander and Krugman (1983) and Krugman (1979, 1980).

Introduction

Intra-industry trade (IIT) is one of the most frequently mentioned stylized facts concerning the contemporary international trade patterns. IIT refers to two-way trade in similar products. This kind of trade occurs mostly between developed countries. A vast theoretical and empirical literature has emerged on this issue. The first studies on IIT by Balassa (1966), Grubel (1967) and Grubel and Lloyd (1975) were of empirical nature. Since Grubel and Lloyd's documentation of an extensive amount of IIT among industrialized (and similarly endowed) countries seemed at odds with the traditional theories of comparative advantage, they provided motivation for the development of the so-called 'New Trade Theory' (NTT) under imperfect competition and increasing returns to scale (IRS).

The NTT has supplied us with two types of theoretical models, commonly categorized into 'large numbers' and 'small numbers' explanations of IIT. The terms 'large' and 'small' refer to the number of firms under the respective market structures. The 'large numbers' explanation refers to the well established theory of trade under monopolistic competition. The pioneering theoretical articles of IIT in differentiated products go back to Dixit and Norman (1980), Krugman (1979; 1980; 1981), Lancaster (1980), and Helpman (1981).¹ More recently, this strand in the literature introduced also firm heterogeneity in terms of labor productivity. This line of research was initiated by Melitz (2003) and subsequently extended by Helpman *et al.* (2004) to include the choice between exporting and horizontal foreign direct investment (FDI).²

The 'small numbers' model, initially proposed by Brander (1981), has provided us with a homogenous product partial equilibrium explanation of intra-industry trade in segmented duopolistic markets. In Brander's model, intra-industry trade arises from firms' incentives to capture some of foreign monopoly rents. Although the essence of the model that firms' exports are the result of their profit motives and that, consequently, trade increases

¹ For the survey of this early literature see: Helpman and Krugman (1985).

² For the survey of this literature see: Helpman (2006).

competition, seems to capture an important aspect of international trade, the Brander model has received relatively little attention in the discussions on intra-industry trade.³ The most notable extensions of this model include theoretical studies of IIT by Brander and Krugman (1983) and Bernhofen (1999) that still used partial equilibrium frameworks. The partial equilibrium oligopoly approach has been also frequently used in studying the choice between exporting and FDI. The examples of such studies include, *inter alia*, Horstmann and Markusen (1987), Sinha (2010), Collie (2011), Cieřlik and Ryan (2012) and more recently Cieřlik (2015a,b; 2016).

This partial equilibrium approach is very typical for the industrial organization (IO) literature, while it is rather uncommon for the international trade literature. In the IO literature it is quite natural to use partial equilibrium frameworks in order to understand the operation of a specific market, at the same time ignoring the wider context. However, in the international trade literature many of the key issues involve comparisons between sectors and links between goods and factor markets. A better understanding of these issues requires the use of a full general equilibrium framework that allows explicitly modeling the links between various markets.

Unfortunately, embedding oligopoly models into a general equilibrium framework has been generally perceived as difficult. This is due to the fact that in such a framework firms should be required to solve general equilibrium problems, while still playing strategically against each other. This poses several technical problems and generally involves quite complex modeling.⁴ Some of these problems have been addressed simply by ignoring them, however, this approach has not met with universal approval.⁵ The recent advances in the theory of international trade using the oligopolistic competition framework can be found in Leahy and Neary (2013), where both quantity and price competition are explored, and strategic interactions between domestic and foreign firms are studied, however, without the endogenization of the number of firms.

³ Due to the extension of the model by Brander and Krugman (1983), which lead to the well-known 'reciprocal dumping' model, the model has had a much bigger influence on the dumping literature than on the intra-industry trade literature.

⁴ For example, as shown by Roberts and Sonnenschein (1977) if oligopolists anticipate the effects of their choices on national income then their reaction functions may behave badly and the equilibrium may not exist. Another problem reported by Gabszewicz and Vial (1972) is that when oligopolists anticipate their impact on the aggregate price level the effects of their actions become sensitive to the deflator employed to evaluate the real value of their profits.

⁵ Examples of this approach include the theoretical studies by Markusen (1984) and Ruffin (2003).

Most recently, Neary (2016) has presented a full general equilibrium model of trade using oligopoly. He assumes continuum-Pollak preferences, which allows to aggregate continuum of sectors in which a small number of domestic and foreign firms operate under Cournot competition. This model does not, however, combine the oligopoly with the monopolistic competition as will be done here.

In this paper, we construct a model of intra-industry trade in oligopolistic markets which is an extension of the seminal model of Brander (1981). We study the role of strategic interactions in intra-industry trade using the generalized Cournot oligopoly framework with many firms. This paper is closely related to the early papers by Brander (1981) and Brander and Krugman (1983) who used a simple partial equilibrium Cournot duopoly.

In contrast to those papers, in our paper we use a more general approach. In particular, we use a full general equilibrium Cournot oligopoly framework with many firms. Moreover, we endogenize the number of firms and study closed and open economy equilibria. Therefore, this paper can be regarded as the extension of the early papers by Brander (1981) and Brander and Krugman (1983). The organization of this paper is as follows. The next section discusses the key assumptions and the model setup. The following one discusses the open economy equilibrium and analyzes the volume of trade. The last section summarizes and concludes.

Model setup

In this section, we discuss the key assumptions of the model. In particular, we assume that there are two countries: Home and Foreign, which represents the rest of the world. Moreover, we assume that the only factor of production in this model is homogenous labor. Finally, we assume that there are two sectors in the economy. We assume that each firm in each sector in each country maximizes its profit.

Demand side

We assume that a representative consumer consumes two classes of goods: differentiated product X , which has many potential specifications, so that there are many varieties of this good, and homogeneous product Y with a single specification. Consumers have identical, homothetic preferences over these two classes of goods, as well as among varieties of differentiated good. Such preferences can be represented by the standard two-tier utility

function of the Cobb-Douglas-Spence-Dixit-Stiglitz (C-D-S-D-S) form, where the upper-tier utility function takes the Cobb-Douglas form, and the lower-tier utility function takes the standard Spence-Dixit-Stiglitz form. Therefore, the consumer utility maximization problem can be written as follows:

$$\max U = C_x^\mu C_y^{1-\mu}, \quad (1)$$

subject to a budget constraint:

$$P_x C_x + P_y C_y = E,$$

where:

$$P_x = \left[\sum_{i=1}^{N_x} (p_x^i)^{\frac{\theta}{\theta-1}} \right]^{\frac{\theta-1}{\theta}},$$

$$C_x = \left[\sum_{i=1}^{N_x} (c_x^i)^\theta \right]^{\frac{1}{\theta}},$$

$$\theta = \frac{\sigma-1}{\sigma} \text{ and } \mu \in (0,1),$$

c_x^i – represents consumption of i -th variety of differentiated good X , C_y – represents consumption of homogenous good Y , N_x is the number of available varieties of good X , P_x is the aggregate price index for the differentiated good X , p_x^i is the price of the i -th variety of good X , P_y is the price of homogenous good Y , and E represents the level of aggregate consumer spending which is equal to the aggregate wage bill.

The upper-tier utility function implies constant expenditure shares μ and $1 - \mu$ on differentiated and homogeneous goods, respectively, and the lower-tier utility function implies the constant elasticity of substitution between any two varieties of differentiated good equal $\sigma = \frac{1}{1-\theta} > 1$. The assumption of homotheticity of preferences permits decomposition of the consumer utility maximization problem into two stages: the allocation of aggregate expenditure between two classes of goods, and the allocation of aggregate expenditure devoted to differentiated good X among different varieties of this good.

The solution of the consumer utility maximization problem (1) given the aggregate expenditure level E , prices p_x^i , P_x , P_y and the number of available varieties N_x yields demand functions for a particular variety of differentiated good X and homogeneous good Y , given by (2) and (3):

$$c_x^i = \mu E (p_x^i)^{-\sigma} P_x^{\sigma-1}, \quad (2)$$

$$C_y = (1 - \mu) \frac{E}{P_y}. \quad (3)$$

Supply side: differentiated product

The differentiated good X is produced in many varieties under increasing returns to scale (*IRS*) and Chamberlinian monopolistic competition using a single factor of production — labor. For simplicity, following the original papers by Krugman (1979, 1980, 1981), the production technology is assumed to be exactly the same for all varieties and all locations. Production of a variety requires a fixed setup cost of F_x expressed in terms of labor and a constant marginal cost m_x also expressed in terms of labor. Hence, the total cost of producing a quantity x of any variety at a given location can be expressed as follows:

$$TC(x) = (F_x + m_x x)w, \quad (4)$$

where w denotes the wage rate. The representative firm producing i -th variety of the differentiated product X maximizes the following profit function:

$$\pi_x^i = p_x^i(x_i)x_i - (F_x + m_x x_i)w. \quad (5)$$

The first order condition for the maximization of (5) yields the standard optimal pricing formula. The price of a representative variety is determined as the constant markup $\sigma/(\sigma - 1)$ over the value of the marginal cost of production, $m_x w$, which is the same for all firms:

$$p_x^i = p_x = \frac{\sigma}{\sigma - 1} m_x w. \quad (6)$$

It can be noted that the equilibrium price of a representative variety of good X depends negatively on the elasticity of substitution σ between particular varieties of good X and positively on both m_x and w . Since the value of σ is greater than 1, the price exceeds the marginal cost and falls to this value only when the elasticity becomes infinite (as it is in the limit case of perfect competition with the infinite number of firms).

Moreover, we assume free entry and exit of firms into the industry in response to profits and losses, respectively. This allows us to determine the break-even level of output x . Substituting (6) into (5) and setting the equilibrium profit to zero yields:

$$x = \frac{F_x(\sigma - 1)}{m_x}. \quad (7)$$

Consumption of each variety per consumer is given by:

$$c_x = \frac{F_x(\sigma - 1)}{Lm_x}, \quad (8)$$

where L is the total labor supply. It can be noted that the amount of output of a representative variety of good X produced depends positively on the ratio of the fixed to marginal cost of production (F_x/m_x), as well as the elasticity of substitution between varieties, σ . The higher the ratio of the fixed to marginal cost of production and the higher the elasticity of substitution between different varieties of the differentiated good the bigger must be the volume of output of the representative variety of the differentiated good.

The number of varieties of the differentiated product that are actually produced can be determined from the good X market clearing condition. The total value of output of all varieties of good X produced in the economy must equal the total value of expenditure falling on good X . Recalling that labor is the only factor of production in this economy, the total expenditure equals the aggregate wage bill.⁶ Hence, the good X market clearing condition can be written as follows:

$$p_x x N_x = \mu E = \mu w L. \quad (9)$$

⁶ This would be true only with free entry also in the second sector where we have oligopoly, which is what we are going to assume.

Therefore, substituting (6) for the equilibrium price of a representative variety and (7) for the break-even level of output and rearranging equilibrium condition (9) yields the equilibrium number of firms producing different varieties of differentiated good X as a function of the constant parameters of the model:

$$N_x = \frac{\mu L}{\sigma F_x}. \quad (10)$$

It can be noted that the number of produced varieties of the differentiated good X increases in the aggregate supply of labor L , the fraction of expenditure devoted to the differentiated good μ , and decreases in the fixed cost F_x of producing the variety and in the elasticity of substitution between varieties σ .

The equilibrium in the market for the differentiated product can be illustrated graphically in Figure 1. One relation between price and consumption in this figure is given by PP curve (the optimal pricing formula) and the second one is given by ZZ (the zero profit condition). The intersection of both curves determines the equilibrium price and consumption.

$$PP: \quad \frac{p_x}{w} = m_x \frac{\sigma}{\sigma - 1}.$$

$$ZZ: \quad \frac{p_x}{w} = \frac{F_x + m_x x}{x} = \frac{F_x}{L c_x} + m_x.$$

It should be noted that an increase in L shifts the ZZ line to the left but leaves PP schedule unchanged. This leaves the equilibrium value of p_x/w unchanged but lowers c_x .

Supply side: homogenous product

The homogenous good Y is also produced under IRS, but under Cournot oligopolistic competition which means that the economies of scale in sector Y are stronger than in sector X . The total cost of producing a quantity y of the homogenous good at a given location can be expressed as follows:

$$TC(y) = (F_y + m_y y)w. \quad (11)$$

The representative firm producing homogenous product Y in the home country maximizes the following profit function:

$$\pi_y^i = p_y^i(Y)y_i - (F_y + m_y y_i)w, \quad (12)$$

where $Y = \sum_{i=1}^{N_y} y_i$ is the aggregate output in the industry. The first order condition for the maximization of (12) yields the optimal pricing formula. The price of the homogenous good Y is determined as the markup over the marginal cost of production:

$$p_y^i = p_y = \frac{N_y}{N_y - 1} m_y w. \quad (13)$$

In contrast to the optimal pricing formula for the differentiated product, now it can be noted here that the markup is not constant and depends on the number of firms that operate in the market for good Y . The greater number of firms increases competition between them and translates into a lower markup. The limiting case of infinitely many firms results in the fall of the price to the value of the marginal cost. As in the case of good X , the equilibrium price of good Y depends positively on both m_y and w .

Similarly as in the case of a differentiated product X , here we also assume free entry (exit) in response to profits (losses). This allows us to determine the break-even level of output of an individual firm producing homogenous good Y . Substituting (13) into (12) and setting the equilibrium profit to zero yields:

$$y = \frac{F_y}{m_y} (N_y - 1). \quad (14)$$

It can be noted that the amount of per-firm output of homogenous good Y produced depends on the ratio of the fixed to marginal cost of production (F_y/m_y) as well as on the number of firms that operate in the market N_y . The higher the ratio of the fixed to marginal cost of production and the greater the number of firms, the larger the volume of output of the homogenous good.

The number of firms producing the homogenous product is endogenous here, and can be determined from the good Y market clearing condition. The total value of output of good Y produced in the economy must equal the total value of expenditure for good Y . Recalling that labor is the only factor of production in this economy, the total expenditure equals the total wage bill. Hence, the good Y market clearing condition can be stated as follows:

$$p_y y N_y = (1 - \mu)E = (1 - \mu)wL. \quad (15)$$

Therefore, substituting (13) for the equilibrium price of good Y and (14) for the breakeven level of output and rearranging equilibrium condition (15) yields the equilibrium number of firms producing homogenous good Y :

$$N_y = \sqrt{\frac{(1 - \mu)L}{F_y}}. \quad (16)$$

It can be noted that the number of firms producing the homogenous good increases in the aggregate supply of labor L , the fraction of expenditure devoted to the homogenous good $(1 - \mu)$, and decreases in the fixed cost F_y . Determination of the number of firms can be presented graphically by showing the price as a function of the number of firms as well as the average cost as a function of the same argument. The optimal pricing formula is given by (13) and shows strictly negative relation of the price with respect to the number of competitors (N_y). As N_y grows, the price approaches the value of the marginal cost ($m_y w$). On the other hand, the formula for average cost can be obtained by using the cost function given in (11):

$$AC = \frac{F_y w}{y} + m_y w = \frac{F_y w N_y}{Y} + m_y w. \quad (17)$$

Since the total expenditure for homogenous good must be equal to total value of its output, it can be shown that:

$$Y = \frac{(1 - \mu)wL}{p_y} = \frac{(1 - \mu)L}{m_y} \cdot \frac{N_y - 1}{N_y}.$$

Substituting the above expression for Y into (17) yields the final formula for average cost which reads:

$$AC = \frac{F_y w}{(1 - \mu)L} \cdot \frac{N_y^2}{N_y - 1} + m_y w. \quad (18)$$

This expression shows that the average cost function is strictly increasing with N_y for all $N_y \geq 2$.

Having obtained the equilibrium number of firms producing the homogenous good, we can now determine the equilibrium price and output. Substituting (16) into (13) yields the equilibrium price of good Y :

$$p_y = \frac{\sqrt{1 - \mu}}{\sqrt{1 - \mu} - \sqrt{F_y/L}} \cdot m_y w. \quad (19)$$

It can be noted that the equilibrium price of good Y depends negatively on the fraction of expenditure devoted to the homogenous good $(1 - \mu)$, and the aggregate supply of labor L , and positively on the fixed cost F_y , the marginal cost m_y and the wage rate w . Substituting (16) into (14) yields the equilibrium per-firm output of good Y :

$$y = \frac{1}{m_y} \left(\sqrt{(1 - \mu)L F_y} - F_y \right). \quad (20)$$

It can be noted that the equilibrium amount of per firm output of homogenous good Y depends negatively on the marginal cost of production m_y and positively on the fraction of expenditure devoted to the homogenous good $(1 - \mu)$, the aggregate supply of labor L , and the fixed cost F_y .

The total output of homogenous good Y can be obtained by multiplying (16) and (20):

$$Y = N_y y = \frac{1}{m_y} \left[(1 - \mu)L - \sqrt{(1 - \mu)L F_y} \right]. \quad (21)$$

It is clear that the total output is an increasing function of the labor supply.

Supply side: allocation of labor

Finally, we discuss the allocation of labor between particular sectors of the economy. The demand for labor in sector producing good X , L_x , can be obtained by multiplying the individual labor requirement by the number of firms that operate in this sector:

$$L_x = (F_x + m_x x) N_x = \mu L. \quad (22)$$

Similarly, the demand for labor in sector producing good Y , L_y can be obtained by multiplying the individual labor requirement by the total number of firms that operate in this sector:

$$L_y = (F_y + m_y y) N_y = (1 - \mu)L. \quad (23)$$

The total supply of labor in the economy must equal the total demand for labor:

$$L_x + L_y = \mu L + (1 - \mu)L = L. \quad (24)$$

Equilibrium in the open economy

For the sake of simplicity, we consider only a two-country case. Suppose the only difference between Home and Foreign is the labor endowment. Home's labor endowment will be denoted by L^h , while Foreign's – by L^f . Open economy (trading) equilibrium is characterized by removing trade restrictions and allowing goods to be sold internationally. Domestic consumers are able to buy foreign varieties of the differentiated product, X , as

well as the homogenous good, Y . The same applies to foreign consumers. As a result of complete trade liberalization, we have a common market effectively.

Trading equilibrium: the differentiated product

By the property of (7), we can immediately see that the per-firm output of the differentiated product does not depend on the size of economy. This leads to the conclusion that opening to free trade in the differentiated product changes neither the per-firm output nor the number of varieties produced in either country. The total number of varieties in the common market will be:

$$N'_x = \frac{\mu(L^h + L^f)}{\sigma F_x} = N_x^h + N_x^f. \quad (25)$$

Per-worker consumption of each domestically produced variety under autarky (pre-trade equilibrium) is:

$$c_x^A = \frac{F_x(\sigma - 1)}{m_x L^h}. \quad (26)$$

Under free trade the equilibrium per-worker level of consumption of a representative variety will be:

$$c_x^T = \frac{F_x(\sigma - 1)}{m_x(L^h + L^f)}, \quad (27)$$

which is obviously smaller compared to the autarky equilibrium. This is because every consumer now spreads their consumption over larger number of varieties but consumes fewer units of each. The same argument holds for both Home and Foreign consumers.

Wage equalization

We can demonstrate that complete trade liberalization results in wage equalization. Home country income in sector X must equal the share of world income spent on domestically produced varieties in that sector:

$$w^h L_x^h = (\mu w^h L^h + \mu w^f L^f) \frac{N_x^h}{N_x^h + N_x^f}. \quad (28)$$

$$\mu w^h L^h = (\mu w^h L^h + \mu w^f L^f) \frac{N_x^h}{N_x^h + N_x^f}. \quad (29)$$

$$w^h L^h \frac{N_x^f}{N_x^h + N_x^f} = w^f L^f \frac{N_x^h}{N_x^h + N_x^f}. \quad (30)$$

By the property of (7) we know that:

$$\frac{N_x^h}{N_x^h + N_x^f} = \frac{L^h}{L^h + L^f}. \quad (31)$$

Therefore:

$$w^h L^h \frac{L^f}{L^h + L^f} = w^f L^f \frac{L^h}{L^h + L^f}. \quad (32)$$

$$w^h = w^f. \quad (33)$$

Trading equilibrium: the homogenous product

The description of the trading equilibrium in homogenous product requires some additional notation. Let us adopt the following:

- y_{hh}^i – sales of i-th domestic firm on domestic market;
- y_{fh}^j – sales of j-th foreign firm on domestic market;
- y_{hf}^i – sales of i-th domestic firm on foreign market;
- y_{ff}^j – sales of j-th foreign firm on foreign market;
- $i = 1, \dots, N_y^h$;
- $j = 1, \dots, N_y^f$.

In the trading equilibrium, the representative domestic firm producing homogenous product Y for both markets maximizes the following profit function:

$$\pi_h^i = p_h(Y_h)y_{hh}^i + p_f(Y_f)y_{hf}^i - \left(F_y + m_y(y_{hh}^i + y_{hf}^i)\right)w, \quad (34)$$

where $Y_h = \sum_{i=1}^{N_y^h} y_{hh}^i + \sum_{j=1}^{N_y^f} y_{fh}^j$ is the aggregate output in the Home market, $Y_f = \sum_{i=1}^{N_y^h} y_{hf}^i + \sum_{j=1}^{N_y^f} y_{ff}^j$ is the aggregate output in the Foreign market, p_h is the price of homogenous good at the Home market, p_f is the price of this good at the Foreign market. For simplicity, we neglect the existence of transport costs. The profit function of a representative foreign firm is given by:

$$\pi_f^j = p_h(Y_h)y_{fh}^j + p_f(Y_f)y_{ff}^j - \left(F_y + m_y(y_{fh}^j + y_{ff}^j)\right)w. \quad (35)$$

Maximization of profits yields the following four F.O.C.:

$$\frac{\partial \pi_h^i}{\partial y_{hh}^i} = \frac{\partial p_h(Y_h)}{\partial y_{hh}^i} y_{hh}^i + p_h(Y_h) - m_y w = 0, \quad (36)$$

$$\frac{\partial \pi_h^i}{\partial y_{hf}^i} = \frac{\partial p_f(Y_f)}{\partial y_{hf}^i} y_{hf}^i + p_f(Y_f) - m_y w = 0, \quad (37)$$

$$\frac{\partial \pi_f^j}{\partial y_{fh}^j} = \frac{\partial p_h(Y_h)}{\partial y_{fh}^j} y_{fh}^j + p_h(Y_h) - m_y w = 0, \quad (38)$$

$$\frac{\partial \pi_f^j}{\partial y_{ff}^j} = \frac{\partial p_f(Y_f)}{\partial y_{ff}^j} y_{ff}^j + p_f(Y_f) - m_y w = 0. \quad (39)$$

By observing the symmetry, we can focus on the Home market only (equations 36 and 38) and then replicate the solutions for Foreign. Let us define the residual supply functions:

$$Y_{hh}^R = Y_h - y_{hh}^i, \quad (40)$$

$$Y_{fh}^R = Y_h - y_{fh}^j. \quad (41)$$

Using these functions we can rewrite the F.O.Cs for the Home market as:

$$p_h(Y_{hh}^R + y_{hh}^i) \left[1 + \frac{\partial p_h(Y_{hh}^R + y_{hh}^i)}{\partial y_{hh}^i} \cdot \frac{y_{hh}^i}{p_h(Y_{hh}^R + y_{hh}^i)} \right] = m_y w, \quad (42)$$

$$p_h(Y_{fh}^R + y_{fh}^j) \left[1 + \frac{\partial p_h(Y_{fh}^R + y_{fh}^j)}{\partial y_{fh}^j} \cdot \frac{y_{fh}^j}{p_h(Y_{fh}^R + y_{fh}^j)} \right] = m_y w. \quad (43)$$

All firms are symmetric, so we can drop the i and j subscripts. Using the inverse demand functions given implicitly by (3), we can represent the two conditions as:

$$p_h \left[1 - \frac{y_{hh}}{N_y^h y_{hh} + N_y^f y_{fh}} \right] = m_y w, \quad (44)$$

$$p_h \left[1 - \frac{y_{fh}}{N_y^h y_{hh} + N_y^f y_{fh}} \right] = m_y w. \quad (45)$$

From these two above it follows that $y_{hh} = y_{fh}$, which states that the individual output at Home market is the same for all domestic and foreign firms. Obviously, the same is true for the Foreign market, where we have $y_{hf} = y_{ff}$. Using these observations, we can solve for the pricing formula which yields:

$$p_y = p_h = p_f = \frac{N_y^h + N_y^f}{N_y^h + N_y^f - 1} m_y w. \quad (46)$$

The pricing formula for foreign market yields identical result, as all firms are assumed to have identical costs. Free entry and exit of firms allows to exploit the zero-profit conditions to find the equilibrium per-firm output:

$$p_y(y_{hh} + y_{hf}) = (F_y + m_y(y_{hh} + y_{hf}))w, \quad (47)$$

$$p_y(y_{ff} + y_{fh}) = (F_y + m_y(y_{ff} + y_{fh}))w. \quad (48)$$

Then the output is given by:

$$y_h = y_{hh} + y_{hf} = \frac{F_y}{m_y} (N_y^h + N_y^f - 1), \quad (49)$$

$$y_f = y_{ff} + y_{fh} = \frac{F_y}{m_y} (N_y^h + N_y^f - 1). \quad (50)$$

These two quantities are the same, which means that all firms have identical per-firm production ($y_h = y_f = y$). The difference between the firms, however, is in the division of their output between the sales ‘at home’ and ‘for exports’. We will return to this issue later. Now, the equilibrium number of firms operating in both markets can be derived from market clearing condition for homogenous product, where the total output by all firms must equal total expenditure by all consumers:

$$p_y(N_y^h + N_y^f)y = (1 - \mu)E^h + (1 - \mu)E^f, \quad (51)$$

from which it follows that the equilibrium number of firms operating on a common market is:

$$N_y^h + N_y^f = \sqrt{\frac{(1 - \mu)(L^h + L^f)}{F_y}}. \quad (52)$$

Given the equilibrium number of firms, the equilibrium per-firm output is given by:

$$y = \frac{1}{m_y} \left(\sqrt{(1 - \mu)(L^h + L^f)F_y} - F_y \right). \quad (53)$$

The exact number of domestic firms and foreign firms can be derived from the labor allocation conditions. Since the production of the differentiated product does not change, nor does the number of firms in sector X , we know that total employment in sector X after the liberalization of trade will remain unchanged. This implies that the share of labor allocated to sector Y is also unchanged. If we assume restrictions in international labor mobility, it implies that the number of firms in each country will be constrained by local labor resources: $N_y^k = (1 - \mu)L^k / l_y^k$, where l_y^k is the per-firm employment required to produce the equilibrium per-firm output in country $k \in \{h, f\}$. It then implies:

$$l_y^k = \sqrt{(1 - \mu)(L^h + L^f)F_y}, \quad (54)$$

and finally:

$$N_y^h = \frac{(1 - \mu)L^h}{\sqrt{(1 - \mu)(L^h + L^f)F_y}}, \quad (55)$$

$$N_y^f = \frac{(1 - \mu)L^f}{\sqrt{(1 - \mu)(L^h + L^f)F_y}}, \quad (56)$$

It implies that the number of firms operating on a common market will be dominated by firms coming from a larger country.

The last thing to determine is the division of per-firm output. The total supply at home must equal demand at home and the total supply at foreign market must equal demand at foreign:

$$N_y^h y_{hh} + N_y^f y_{fh} = \frac{(1 - \mu)E^h}{p_y} = \frac{(1 - \mu)L^h(N_y^h + N_y^f - 1)}{m_y(N_y^h + N_y^f)}, \quad (57)$$

$$N_y^h y_{hf} + N_y^f y_{ff} = \frac{(1 - \mu)E^f}{p_y} = \frac{(1 - \mu)L^f(N_y^h + N_y^f - 1)}{m_y(N_y^h + N_y^f)}. \quad (58)$$

These two conditions together with earlier observations of $y_{hh} = y_{fh}$ and $y_{ff} = y_{hf}$ eventually yield:

$$y_{hf} = \frac{L^f}{L^h + L^f} y_h, \quad (59)$$

$$y_{fh} = \frac{L^h}{L^h + L^f} y_f. \quad (60)$$

These two conditions determine the division of per-firm output between sales ‘at home’ and sales ‘for exports’. It can be noted that the share of sales ‘for exports’ is the function of the relative size of the destination economy. It follows then that firms from a relatively small country will exhibit higher export orientation of their output.

The volume of trade

The volume of exchange in goods X and Y can now be analyzed. Home country exports N_x^h varieties of the differentiated product. The quantity of exports of each variety is proportional to the size of the destination market, and therefore the total exports can be written as:

$$\begin{aligned} EXP_x &= N_x^h \frac{F_x(\sigma - 1)L^f}{m_x(L^h + L^f)} = \frac{\mu L^h}{\sigma F_x} \cdot \frac{F_x(\sigma - 1)L^f}{m_x(L^h + L^f)} \\ &= \frac{\mu(\sigma - 1)}{\sigma m_x} \cdot \frac{L^h L^f}{(L^h + L^f)}. \end{aligned} \quad (61)$$

Similarly, we can write the aggregate imports of N_x^f varieties from Foreign:

$$\begin{aligned} IMP_x &= N_x^f \frac{F_x(\sigma - 1)L^h}{m_x(L^h + L^f)} = \frac{\mu L^f}{\sigma F_x} \cdot \frac{F_x(\sigma - 1)L^h}{m_x(L^h + L^f)} \\ &= \frac{\mu(\sigma - 1)}{\sigma m_x} \cdot \frac{L^f L^h}{(L^h + L^f)}. \end{aligned} \quad (62)$$

The volume of trade is therefore given by:

$$VT_x = EXP_x + IMP_x = \frac{2\mu(\sigma - 1)}{\sigma m_x} \cdot \frac{L^f L^h}{(L^h + L^f)}. \quad (63)$$

Denoting $\bar{L} = L^h + L^f$ we can differentiate the volume of trade with respect to the size of home country, to get:

$$\frac{\partial VT_x}{\partial L^h} = \frac{2\mu(\sigma - 1)}{\sigma m_x} \cdot \frac{(\bar{L} - 2L^h)}{\bar{L}}. \quad (64)$$

The volume of trade is maximized when $L^h = \bar{L}/2$, which implies that $L^h = L^f$ (both countries are of equal size).

The exchange in homogenous good Y involves exporting the quantity y_{hf} by N_y^h firms from Home and importing y_{fh} from N_y^f firms from Foreign. Using (49), (50), (52), (55), (56), (59) and (60) we get:

$$EXP_y = \frac{(1 - \mu)L^h L^f}{m_y(L^h + L^f)} - \frac{L^h L^f \sqrt{F_y(1 - \mu)}}{m_y(L^h + L^f) \sqrt{L^h + L^f}}, \quad (65)$$

$$IMP_y = \frac{(1 - \mu)L^h L^f}{m_y(L^h + L^f)} - \frac{L^h L^f \sqrt{F_y(1 - \mu)}}{m_y(L^h + L^f) \sqrt{L^h + L^f}}. \quad (66)$$

The volume of trade $VT_y = EXP_y + IMP_y$ is again obviously maximized when both countries are of equal size, $L^h = L^f$.

Conclusions

In this paper, we have developed a general equilibrium intra-industry trade model with differentiated and homogenous goods produced under increasing returns to scale. In contrast to earlier studies by Krugman and Brander, we endogenized the number of firms operating on the market by assuming free entry and exit in the industry.

The effects of trade liberalization materialized through the increased number of varieties of the differentiated product which increased consumers' utility (the *love for variety* effect). The other effects of trade liberalization were the increased competition between firms in the homogenous good market, which lowered the markup and the equilibrium price and increased total sales. By assuming free entry, all firms operated at zero profits, which implied positive welfare effects of trade due to increased consumer surplus (even if trade costs were positive, which we have not studied here). However, this effect may not always be the case in the previous theoretical studies with fixed number of firms and the existence of positive trade costs.

The further implication of this model is that when two countries open to trade their trade flows are maximized when they are both of equal size. In the case of unequal country size firms coming from a smaller country would also have a higher share of exports in their per-firm output.

The model presented here can be extended in a number of ways. Firm heterogeneity can be modelled by assuming marginal costs being drawn from a certain distribution. Trade costs can be introduced to the model to allow for trade-offs between exporting and FDI strategy choice of firms.

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Annex

Figure 1. Equilibrium in the market for differentiated product

Figure 2. Equilibrium in the market for homogenous product