Hybrid demand forecasting models: pre-pandemic and pandemic use studies

JEL Classification: M21; M15; C53

Keywords: forecastHybrid; demand forecasting; statistic model; neural networks

Abstract

Research background: In business practice and academic sphere, the question of which of the prognostic models is the most accurate is constantly present. The accuracy of models based on artificial intelligence and statistical models has long been discussed. By combining the advantages of both groups, hybrid models have emerged. These models show high accuracy. Moreover, the question remains whether data in a dynamically changing economy (for example, in a pandemic period) have changed the possibilities of using these models. The changing economy will continue to be an important element in demand forecasting in the years to come. In business, where the concept of just in time already proves to be insufficient, it is necessary to open new research questions in the field of demand forecasting.

Purpose of the article: The aim of the article is to apply hybrid models to bicycle sales e-shop data with a comparison of accuracy models in the pre-pandemic period and in the pandemic period. The paper examines the hypothesis that the pandemic period has changed the accuracy of hybrid models in comparison with statistical models and models based on artificial neural networks.

Models: In this study, hybrid models will be used, namely the Theta model and the new forecastHybrid, compared to the statistical models ETS, ARIMA, and models based on artificial neural networks. They will be applied to the data of the e-shop with the cycle assortment in the period from 1.1. 2019 to 5.10 2021. Whereas the period will be divided into two parts, pre-
pandemic, i.e. until 1 March 2020 and pandemic after that date. The accuracy evaluation will be based on the RMSE, MAE, and ACF1 indicators.

Findings & value added: In this study, we have concluded that the prediction of the Hybrid model was the most accurate in both periods. The study can thus provide a scientific basis for any other dynamic changes that may occur in demand forecasting in the future. In other periods when there will be volatile demand, it is essential to choose models in which accuracy will decrease the least. Therefore, this study provides guidance for the use of methods in future periods as well. The stated results are likely to be valid even in an international comparison.

Introduction

In the academic sphere as well as in business practice, there are still discussions about models of demand forecasting in business economics. These still used statistical models are consequently such a standard solution of many forecasting systems even nowadays. In recent years, models based on artificial intelligence have come to fore, primarily artificial neural networks, in spite of the fact that these models may not always be the best. Practice shows that the training data indicates great accuracy on the test data; this may not be the case (Kolková, 2018, pp. 102–119).

The most modern models used in forecasting were hybrid models, which take advantage of both approaches. As early as 1969, Bates and Granger (2017) introduced combined models and herewith provided the conceptual foundations for the further use of existing models. Hybrid models combine existing methods with artificial intelligence. Not only their mutual combination, but also the possibility of calculating model parameters on a static basis using artificial intelligence.

Initially in M4-Competition (Petropoulos & Makridakis, 2020, pp. 3–6), these models were rated as the most successful. However, it is clear that each model has its justification, and its use depends mainly on the selected data.

This article uses the models that were evaluated as more successful in this competition. And they were hybrid methods that successfully participated in this competition.

As noted earlier, the aim of this article is to apply hybrid models to bicycle sales e-shop data with a comparison of accuracy models in the pre-pandemic period by contrast in the pandemic period. The paper examines the hypothesis that the pandemic period has changed the accuracy of hybrid models in comparison with statistical models and models based on artificial neural networks.

Subsequently, hypotheses were specified to meet the goal. The first hypothesis: that the pandemic period has changed the accuracy of hybrid models compared to statistical models and models based on artificial neural
networks. The second hypothesis then assumes that hybrid models were the most accurate in the pre-pandemic period. The third hypothesis then implies that the most accurate statistical models remained in the pandemic period. To make certain these hypotheses, static models, models based on artificial neural networks, and hybrid models on daily e-shop data in the years 2019–2021 were applied.

Additionally, the literature review will define previous research on hybrid models and the application of models to practical data. Hybrid models will be described in detail in the methodology chapter, and a statistical description of the data will be performed. In the results section, the results of the calculations will be presented in the figures and tables. These were subsequently commented on in the discussions and the achievement of the goal and the acceptance or refutation of hypotheses were evaluated. Finally, the results will be summarized in conclusion.

**Literature review**

Hybrid models represent a combination of models based on artificial intelligence and statistical models (Smyl, 2020, pp. 75–85). In 2018, a large competition of forecasting models was organized. These competitions are very important in the forecasting community. It is currently attended by leading researchers from all over the world. These are not only researchers from the academic sphere, but also researchers from business practice. This makes these competitions even more serious.

The results of the M4 Competition (Makridakis *et al.*, 2018, pp. 802–808) demonstrate that the combination of several prognostic models gives the best results. Of the 17 models evaluated as the most accurate in this competition, 12 were mainly combinations of statistical models.

The biggest surprise of the M4 Competition was the hybrid approach. The hybrid model was also the absolute winner of this competition. The hybrid model in the M4 Competition was presented by Smyl (2020, pp. 75–85), a data scientist from Uber Technologies. Smyl applied a hybrid model based on artificial neural networks and a statistical model inspired by exponential smoothing. He used exponential smoothing formulas to deseasonize and normalize time series and used advanced neural networks to extrapolate.

Hereafter the second most accurate model in this competition was one that combined seven statistical and one machine learning model. The weights here were calculated using a machine learning algorithm. This model was presented by the Spanish University of A Coruña and the Aus-
talian Monash University (Montero-Manso et al. 2020, pp. 86–92). Another new hybrid model was developed by Nontapa et al. (2021), the model uses the SARIMAX prediction model and an artificial neural network.

The results of the M5Competition (Makridakis et al., 2022) are currently being published. This competition focused on forecasting retail sales, so it was very close to demand forecasting, which is also applied in this article. However, not all results are currently available yet.

Another important forecasting competition is provided by Kaggle forecasting competitions (Bojer & Meldgaard, 2021). Here it follows that models using cross-learning tend, decision trees and neural networks are strong prognostic models. As a result, in this study we will supplement neural networks with other models and by creating hybrid models we want to examine their usability.

In this context, it is worthwhile to consider an interesting study on hybrid models in a study of Zougagh et al. (2021). In this study, contributions to hybrid prediction models since 2005 published in Scopus databases have been analysed; IEEE; ERIC; Google scholar. A total of 70 articles dealing with hybrid forecasting models were analysed. The results show that interest in hybrid models has grown significantly since 2016. Artificial neural networks are most often included in hybrid models, which were in 41% of models. They were also applied in 20% autoregressive models (AR, ARIMA or ARMA). In 15%, the authors of hybrid models used the Genetic algorithm and the support vector machine was used in 13% and fuzzy systems 11%.

Furthermore, they dealt with hybrid models directly in demand forecasting (Siddiqui et al., 2021, pp. 1–11). A parallel series hybridization of seasonal intelligent-based statistical model for demand forecasting (Bahrami & Khashei & Aminidoust, 2021) and a selected hybrid model used for demand forecasting have been used (e.g. Zhang et al., 2021; Kolková & Ključnikov, 2021, pp. 1063–1094).

At the same time, there are still discussions about whether models based on artificial intelligence or statistical models are more accurate (Kolková, 2020, pp. 90–105). Spiliotis et al. (2022) state that some machine learning models provide better predictions, both in terms of accuracy. He came to similar conclusions to Cuhadar (2020, pp. 55–70), who forecasted tourism in Croatia and concluded that models based on artificial neural networks always work more accurately than statistical models. Similarly to Pereira and Cerqueira (2021, pp. 1–18) in forecasting the demand for hotel services, he established that the use of machine learning models can reduce the mean square error by up to 54% for the 1-day forecast horizon and by up to 45% compared to traditional models of exponential smoothing for a 14-day
forecast horizon. Balaji Prabhu and Dakshayini (2020, pp. 35–47) argue that the multiple linear regression model and the artificial neural network model are both useful, reliable, and relatively effective tools for optimizing the effects of demand prediction in food harvest supply management to satisfactorily match social needs. The study by Abbasimehr et al. (2020, pp. 345–366) compared artificial intelligence models with statistical ones. The LSTM multilayer network model was the most accurate, the second most accurate model was artificial neural networks, however, in the third place it was marked as the most accurate ETS model.

Furthermore, scientists and especially practitioners evaluate whether the most accurate models are always the best for practical use (Kolková & Navrátil, 2021, pp. 123–141). Especially due to the computational demand of the prediction or the financial cost of maintaining such a model in real business operations. In Kolková and Navrátil (2021, pp. 123–141) parameters other than accuracy were also examined. The results of this study illustrated that models based on deep learning have proven to be the worst on runtime and computing demand.

On the other hand, the authors address the issue whether or not the differences between the accuracy of models based on artificial intelligence and statistical models are related to the time series studied. Foldvik Eikeland et al. (2021) conclude in their study that statistical models achieve higher accuracy over longer prediction horizons compared to neural networks, while machine learning approaches work better in predicting loads at shorter time intervals. On the contrary, Marčeš (2019, pp. 317–322) showed that all the models he applied (ARIMA, NN, SVM) are suitable as prediction models for use in prognostic systems that commonly predict the values of variables in competitive energy markets. Bui et al. (2020, pp. 382–406) recommend new models of data filtering to increase the accuracy of models based on artificial neural networks. Sudden changes in demand can occur, especially in data from developing countries. The results confirm that the accuracy of these models can be significantly improved by virtue of the proposed model of statistical data filtering.

Time-series research, which is affected by external shocks, is still an important element of demand forecasting. Principally in the pandemic period, demand may have developed abnormally. In this study, data are distributed and the most accurate prognostic model is selected in a period of stable development and in a period of economic shocks caused by a pandemic. Research is not yet focused on this area.

Data on bicycle sales in the Czech Republic during the pandemic were selected for this study. Only a few scientific works deal with this issue. The results of Habib and Anik (2022) showed that bicycle sales increased sig-
significantly in the pandemic, while car sales decreased. The same logic underlies an article by Ramírez et al. (2021) where the authors found, via a machine learning model, that there was a 29.76% decrease in sales in the automotive industry in the Mexican market. Studies comparing online sales also provide interesting results. According to Fairlie and Fossen (2022), it is possible to distinguish the fields affected by the pandemic, such as accommodation facilities, which lost up to 91% of taxable sales in the place of study, on the contrary, online sales of goods increased up to 180%. Wang et al. (2020) declare how the pandemic affected the change in sales style in the dairy industry. For the time being, the authors have not dealt with the analysis of methods suitable for quantifying the demand for bicycles yet.

**Research methods**

We illustrate this procedure for this study by using data from the online store with bicycle sales. The data declares purchases via the e-shop. The data set contains 878 values of daily revenues and daily number of e-shop orders for the years 2019, 2020 and part of the year 2021. Exactly, there is from 1.1.2019 to 8.9.2021. The descriptive characteristics are defined in Tab. 1.

These data are chosen mainly because there has been a spike in demand for this product due to the pandemic. Such a change in consumer preferences is very likely to affect the degree of accuracy of demand forecasting models as well. Also, the online distribution channel is currently becoming one of the important distribution channels. During the pandemic period, this distribution channel also spread among small and medium-sized enterprises, and its further use can be expected in the future.

The estimated Hurst exponent of the affected variables, shown in Tab. 1, is significantly greater than 0.5. This suggests that the series is not random and is controlled by persistent patterns (the Hurst exponent is an indicator of randomness). If this is not accidental, the use of prediction models is justified.

The minimum daily turnover and the number of orders were 0. The maximum daily turnover was CZK 2,371,324 and the maximum number of orders placed per day was 330. The data on mean and median are also shown in Tab. 1.

The data were decomposed using additive decomposition. Fig. 1 shows the breakdown of daily turnover. This chart clearly shows an increase in the time of the pandemic. However, the seasonality of the demand for these
goods remains the same. Fig. 2 defines the breakdown of the daily number of orders, where a similar progress is evident.

The study verifies three hypotheses. With these hypotheses, we want to verify whether the pandemic affected the forecasting possibilities in terms of the accuracy of hybrid models in contrast to statistical models and models based on artificial neural networks.

**H1:** *The pandemic period has changed the accuracy of hybrid models compared to models based on standard statistical indicators (hereafter statistical models) and the neural network model.*

**H2:** *In the pre-pandemic period, hybrid models were the most accurate.*

**H3:** *In the pandemic period, the most accurate models are statistical.*

Prior research has suggested each model is evaluated using accuracy. The RMSE (root mean square error) coefficient was used to calculate accuracy. This is defined by Hyndman and Athanasopoulos (2018), and can be described by formulas (1) and (2),

\[ RMSE = \sqrt{MSE}, \]

where RMSE has the same unit as the original time series. The MAE indicator is also declared in the same units, which expresses the average deviation of the actual values from the forecasted ones. The median can also be used instead of the mean deviation. We then describe both calculations as follows,

\[ MAE = \frac{1}{M} \sum_{p=n+1}^{N} |y_p - \hat{y}_p| \text{ or } MAE = mean|e_t| \]

The last accuracy used is ACF1. This indicator is defined as the autocorrelation of delay errors 1. In essence, it expresses the degree to which the current value is affected by previous values in a time series. The autocorrelation function for a delay of length k can be defined as

\[ \rho_k = \frac{y_k}{\gamma_0} = \frac{\gamma_k}{\sigma_y^2} \]

Mentioned above, ETS, ARIMA, models based on artificial neural networks, are used in this work. These will be compared with two hybrid
models. The first is the Theta model, the second is the forecastHybrid model.

The deterministic model ETS (Error, Trend, Seasonal) according to Brown (1959); Holt (2004); Winters (1960) can be defined using formulas (4), (5), (6) and (7),

\[ y_t = l_{t-1} + b_{t-1} + s_{t-m} + \epsilon_t, \]  

where

\[ l_t = l_{t-1} + b_{t-1} + \alpha \epsilon_t \]  

\[ b_t = b_{t-1} + \beta \epsilon_t \]  

\[ s_t = s_{t-m} + \gamma \epsilon_t, \]

where \( l_t \) is estimate level, \( b_t \) are estimated trend, \( t \) and \( s_t \) express the seasonality, \( \alpha, \beta \) and \( \gamma \) are weight coefficients.

ARIMA (AutoRegressive Integrated Moving Average) is defined in (Box & Jenkins, 1976). ARIMA models use the Box-Jenkins model. The ARIMA model with the seasonal component can be expressed in the form ARIMA \((p, d, q)(P, D, Q)\), where \( p \) represents the degree of the autoregressive part, \( d \) the degree of differentiation, and \( q \) the degree of the moving average part. Formulas can be used to calculate ARIMA (7) and (8),

\[ \varphi(B)w_t = \theta(B)\epsilon_t, \]  

where

\[ w_t = \Delta^d y_t \]

This model can also be described in a special form, ARFIMA (AutoRegresive Integrated Moving Average). According to this model (Box & Jenkins, 1976), it is possible to estimate and determine the correlation in stationary processes even for very distant random variables. ARFIMA in this article is defined by the relationship (9),

\[ \varphi(B)(1 - B)^d y_t = \theta(B)\epsilon_t \]
**Artificial neural networks** are inspired by processes in the human brain. The output of a neuron is calculated when the sum of the inputs to the neuron $x_i$ multiplied by their specific weights $w_{j,i}$ exceeds a certain value, which we call the distortion. The neuron can be described in this way

$$y_j = f\left(\sum_{i=1}^{m} x_i \cdot w_{j,i} - b_j\right),$$

where $x_i$ is the specific value on the $i$-th input, $w_{j,i}$ are the weights of this input, $b_j$ is bias, $m$ is the total number of inputs, $f$ is a transformation function, and the output values of $y$. Process artificial neural networks is shown in Figure 3. In this article, the Nnar(p,k) model is used, where k is the number of hidden nodes. The inputs are for lags 1 to $p$.

This model tends to be very accurate on training data, but is usually less accurate on test data. However, it usually achieves good accuracy.

The first selected hybrid model *forecastHybrid* is defined in Shaub and Ellis (2020). This model combines ARIMA, ETS, and Theta, Tbats (the name this model is an acronym denoting its salient features T for trigonometric regressors to model multiple-seasonalities, B for Box-Cox transformations, A for ARMA errors, T for trend and S for seasonality, and artificial neural networks). The whole *forecastHybrid* model is then defined by formula (11),

$$y(i) = \sum_{m=1}^{n} w_m \cdot f_m(i),$$

where $w_m$ is weight to each $m$ of the $n$ model, $f_m(i)$ individual model for time horizon $i$. This model is called *forecastHybrid* and is published in a package in the statistical program R (Shaub & Ellis, 2020).

The Theta model was created by Assimakopoulos and Nikopoulos (2000, pp. 521–530). The model is based on the concept of modification of local fluctuations of the time series using the coefficient Theta (denoted by the Greek letter $\theta$). This coefficient is applied to the second difference of the time series, in fact according to the relation,

$$X_{new}''' = \theta \cdot X_{data}'',$$

where

$$X_{data}''' = X_t - 2 \cdot X_{t-1} + X_{t-2}$$
Preceding all others, the initial time series is divided into two or more Theta-lines, and each is then extrapolated separately. The predictions are then simply combined. A different combination of Theta-lines can be used for each forecast horizon.

Of course, hybrid models have their advantages and disadvantages. The `forecastHybrid` model can be a great advantage for analysts and businesses that need to forecast a large number of time series automatically. However, the model can certainly be further expanded and improved. In particular, it is the use of alternative options in the form of selecting the weights of individual models in `forecastHybrid`.

The Theta model is also constantly being developed. Further development of this model assumes that more than a two-line model will be applied, but it is currently uncertain whether the use of more lines will lead to more accurate results. Besides another development option is to use different scales when combining Theta-lines. Further research now concerns the use of different Theta-lines according to the qualitative and quantitative characteristics of a given time series.

The models are applied using automatic functions and auto-tuning in R and Python. The packages `forecast`, `fpp3`, `fable`, `forecastHybrid` are applied in the R language. Automation of calculations enables the use of methods even in companies that do not have high computing power and specialized workers. For this reason, automatic functions without manual tuning are taught in this work as well.

## Results

The ETS model is defined in this article according to the automatic function of the `Forecast` package in R language. The most suitable combination of parameters was selected from this automatic function. The most suitable model for all forecasts was simple exponential smoothing with additive errors.

According to the ETS model, the model is clearly more accurate before a pandemic. The model predicted stagnation and failed to predict a short-term increase in demand at the beginning of the pandemic period. The results of the model can be seen in Fig. 3. The accuracy is then recorded in Tab. 2.

Overall, the ARIMA model was again calculated on the basis of the automatic function, namely `auto.arima`. The best ARIMA models have been selected according to either AIC, AICc or BID values. As consistent with this automatic calculation, they were selected as the most suitable models
with different parameters. For the period during the pandemic, the parameters were ARIMA (1,1,2), for the period before the pandemic, the models differed, and for the revenue forecast, it was ARIMA (0,1,1) for the number of orders ARIMA (2,1,1). The results are shown in Fig. 4. The accuracy of each model is shown in Tab. 3.

The accuracy results confirm that the accuracy of this model in the pre-pandemic period was higher than in the pandemic period. These conclusions underwrite both the accuracy of turnover and the number of orders. All models predict stagnation of future development. The sharp short-term increase caused by the ARIMA pandemic could not be predicted by the pre-pandemic model.

The nnetar function was used to calculate the forecast based on artificial neural networks. By all means, this again finds the most suitable parameters for artificial neural networks. These are defined by NNAR \((p, k)\), where \(p\) is the optimal number of lags and \(k\) is the number of hidden nodes. For the revenues forecast in the post-pandemic data, the NNAR \((26.14, 1)\) was the most suitable, for the other data it was the NNAR \((14.8, 1)\), as can be seen in Fig. 5. Accuracy results are defined in Tab. 4.

For models based on artificial neural networks, the results declare that the models showed greater accuracy in the pre-pandemic period. The prognosis principle is somewhat different according to artificial neural networks and they predict the completion of the started trend; however, they could not predict a sharp increase in the period at the beginning of the pandemic.

As a matter of fact, in the next part of the calculations, the results of hybrid models have already been quantified. The first of all was hybridModel. The second model was the Theta model. Both models were calculated in the basic settings of the model authors. The results can be seen in Fig. 6 and the accuracy measures are in Tab. 5.

Even according to hybrid models, the results confirm that the model was more accurate in the period before the pandemic. During the pandemic period, the error rate increased. This hybrid model already forecasts a slight increase in demand in the pre-pandemic period. We can expect a decrease according to this model in the following period.

The last Theta model confirms the previous results, as we can see in Fig. 7 and the model is also more accurate in the pre-pandemic period, see Tab. 6. It also confirms the future stagnant trend in demand.

All the methods which were utilized showing a decrease in accuracy and react significantly to a change in the market environment. Minor fluctuations can be seen in the demand forecast using the number of orders, see Table 7. Model forecastHybrid performance dropped by 70.07% (respectively 21.6% in the number of orders) during the pandemic, measured using
AFC1. This is the lowest drop. forecastHybrid is the most robust and immune to the sudden structural changes.

**Discussion**

The study verified three hypotheses. The hypotheses were based on influencing or not influencing the possibilities of pandemic forecasting in terms of the accuracy of hybrid models. They were considered in contrast to statistical models and models based on artificial neural networks.

Hypothesis H1 was confirmed by a study in the pandemic period, which was characterized by large changes in demand, the accuracy of all models was worse. Hybrid models, as well as statistical models based on artificial neural networks, showed poorer accuracy for both turnover and order count forecasts. Neither hybrid model can say that a pandemic affects model accuracy less than statistical models based on artificial neural networks.

Hypothesis H2 was acknowledged in the pre-pandemic and in the pandemic period was the most accurate forecastHybrid model. This model was the most accurate for both turnover-based data and order numbers. Thus, it was confirmed that hybrid models have their justification and are suitable for wider application in practice.

As follows from the previous text, hypothesis H3 was not confirmed and even in the dynamic pandemic period, the hybrid model, namely forecastHybrid, emerged the greatest accuracy. All results are summarized in the Tab. 8. Here, you can see the order of the models according to accuracy.

Tab. 8 shows that forecastHybrid was the most accurate model in all data types. Surprisingly, the ARIMA statistical model is marked as the second most accurate. The second hybrid model, the Theta model, was up to 3, respectively, 4 in accuracy.

Since the forecastHybrid model is a new model published in 2020 (Shaub & Ellis, 2020), there are not many studies on this topic yet. A study based directly on the demand forecast can be found in Kamboj et al. (2020, pp. 63–82). In the article by Atıcı and Pala (2022, pp. 3293–3312) we find similar results, and the forecastHybrid model is one of the most accurate. Even in this study, the results differed according to the data used.

Thus, the hybrid models did not answer the question of which forecasting model is the most accurate of all the data and for all the measures of accuracy. However, in this study, they showed that their accuracy even in the pandemic period has a justification.
Given these points, demand forecasting is currently a very debated topic. Dynamic changes in the business environment, caused first by the Covid-19 pandemic and then by the war conflict in Ukraine, show that the popular concept of just in time is no longer suitable. Babai et al. (2022) confirm that the classic purchasing strategy often leads to higher inventory costs and emphasize the importance of forecasting lead-time demand variance. This concept may be an interesting addition to this study.

Our study deals with a continuous demand, however, in practice there is also an intermittent demand with a high percentage of zero values. According to Tian et al. (2021), a combined method is suitable for this type of data. As in our study, the classical exponential smoothing model was not the most appropriate. Similar models involved here were also used by Siddiqui et al. (2022). ARIMA, etc, Theta models were also applied here. The application was implemented on the data of a pharmaceutical company, which in the relevant period also showed high volatility, the same as our data. The authors came to the same conclusion about the suitability of using a hybrid model (in this case ARHOW).

Zhang and Lu (2022) chose a different type of data, namely data from the hotel industry, which, on the other hand, had a decreasing tendency during the pandemic. In this case, it is not enough to select prognostic methods and evaluate their accuracy. Our conception would be insufficient here. As has been noted, according to the authors, it is necessary to supplement the forecasts with scenarios describing recovery patterns.

As already shown in the previous text, the weakest point of this study is its focus on one type of data. Nevertheless, by supplementing other studies and combining their results, a comprehensive overview can be created. Of course, we will continue to study and follow up on this issue. Above all in the sense of expanding different types of data. A similar topic is already covered by Ulrich et al. (2022). This article also confirms our assumption that it is unlikely that a single forecasting model could be found that would have the best accuracy across all products of retail demand.

To take into consideration, a different approach to investigating demand forecasting was chosen by Fildes and Goodwin (2021). The authors point out that little research examines how forecasting systems based on advanced statistical methods are actually used in businesses. They came to the conclusion that, despite the accuracy of the models, non-normative forecasting practices are constantly added.
Conclusions

The aim of this article was to apply hybrid models to the data of the cycle assortment e-shop with a comparison of the accuracy of models in the pre-pandemic period and in the pandemic period. The article tested three hypotheses. Hypothesis H1 was confirmed by the study in the pandemic period; the accuracy of all models was worse. Hypothesis H2 was confirmed and the most accurate model in all data and in both pre-pandemic and pandemic periods was the most accurate hybrid model. On the other hand, the H3 hypothesis was not confirmed and the results showed that even in the dynamic pandemic period, the model from the group of hybrid models showed the greatest accuracy.

To meet this goal, two hybrid models have been identified, namely the `forecastHybrid` from Shaub and Ellis (2020) and the Theta model (Assimakopoulos & Nikolopoulos, 2000), which is one of the older hybrid models. From the statistical models, the ETS and ARIMA models were selected for comparison in the form of an automatic calculation `auto.arima`. Another comparison was with the model of artificial neural networks.

The `forecastHybrid` model was created for Uber Technologies, and its practical use in this company is therefore already implemented. In this study, the model was shown to be suitable for other companies and for other data, even for pandemics. As the model was created only recently, its wider application will only be realized in practice.

The main limitations of this study are, of course, the application to the data of one type of business. As already mentioned in the discussion chapter, research in this area should be continued, as evidenced by another study by the authors (Kolková et al., 2022). Using forecasting methods during periods of demand stability and low volatility is easy. The results of such studies can then be misleading and lead businesses to falsely believe that forecasting models are perfect. However, investigating the accuracy of models in a period of dynamic changes in the economy is currently a topic that is little explored. It is necessary to open up these areas of research and offer companies appropriate models to support their decision-making.

In future research, it would be appropriate to extend the studies to data from companies with different business topics. It will also be stimulating to verify if the accuracy of the models has returned to normal in the aftermath of the pandemic. This, of course, is only possible over time. It would then be appropriate to define whether demand will always return to its original level, even in the modern economy.
References


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Annex

Table 1. Statistical description

<table>
<thead>
<tr>
<th>Data</th>
<th>Min.</th>
<th>Max.</th>
<th>Mean</th>
<th>Median</th>
<th>Standard deviation</th>
<th>Hurst exponent</th>
</tr>
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<tbody>
<tr>
<td>Revenues</td>
<td>0 Kč</td>
<td>2 371 324 Kč</td>
<td>260 993 Kč</td>
<td>168 619 Kč</td>
<td>296 840.9421</td>
<td>0.9378</td>
</tr>
<tr>
<td>Number of orders</td>
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<td>330</td>
<td>64</td>
<td>46</td>
<td>56.3552</td>
<td>0.9591</td>
</tr>
</tbody>
</table>

2019

| Revenues           | 0 Kč   | 918 742 Kč | 138 705 Kč | 102 916 Kč | 131 155.9072       | 0.8812         |
| Number of orders   | 0 Kč   | 118      | 35      | 31       | 23.1490            | 1.0096         |

2020–2021

| Revenues           | 0 Kč   | 2 371 324 Kč | 330 113 Kč | 232 863 Kč | 339 065.7547       | 0.8597         |
| Number of orders   | 0 Kč   | 330      | 81      | 63       | 62.4749            | 0.8812         |

Table 2. Accuracy results according to the ETS model

<table>
<thead>
<tr>
<th>data</th>
<th>RMSE</th>
<th>MAE</th>
<th>ACF1</th>
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<tbody>
<tr>
<td>Revenues forecast: pre-pandemic data</td>
<td>123660.7</td>
<td>91029.67</td>
<td>0.0251</td>
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<td>Revenues forecast: post-pandemic data</td>
<td>220112.3</td>
<td>158069.3</td>
<td>0.1134</td>
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<td>Number of orders: pre-pandemic data</td>
<td>20.2868</td>
<td>15.7006</td>
<td>0.0027</td>
</tr>
<tr>
<td>Number of orders: post-pandemic data</td>
<td>37.4864</td>
<td>27.7062</td>
<td>0.1247</td>
</tr>
</tbody>
</table>

Table 3. Accuracy results according to the ARIMA model

<table>
<thead>
<tr>
<th>data</th>
<th>RMSE</th>
<th>MAE</th>
<th>ACF1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenues forecast: pre-pandemic data</td>
<td>123645.3</td>
<td>90951.2</td>
<td>0.0243</td>
</tr>
<tr>
<td>Revenues forecast: post-pandemic data</td>
<td>218015</td>
<td>158084.5</td>
<td>0.0020</td>
</tr>
<tr>
<td>Number of orders: pre-pandemic data</td>
<td>20.2256</td>
<td>15.7006</td>
<td>0.0027</td>
</tr>
<tr>
<td>Number of orders: post-pandemic data</td>
<td>37.0503</td>
<td>27.5582</td>
<td>0.0007</td>
</tr>
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</table>

Table 4. Accuracy results according to the neural network model

<table>
<thead>
<tr>
<th>data</th>
<th>RMSE</th>
<th>MAE</th>
<th>ACF1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenues forecast: pre-pandemic data</td>
<td>123660.7</td>
<td>91029.67</td>
<td>0.0251</td>
</tr>
<tr>
<td>Revenues forecast: post-pandemic data</td>
<td>220112.3</td>
<td>158069.3</td>
<td>0.1134</td>
</tr>
<tr>
<td>Number of orders: pre-pandemic data</td>
<td>20.2868</td>
<td>15.8433</td>
<td>0.0650</td>
</tr>
<tr>
<td>Number of orders: post-pandemic data</td>
<td>37.4864</td>
<td>27.7062</td>
<td>0.1247</td>
</tr>
</tbody>
</table>
Table 5. Accuracy results according to the *forecastHybrid*

<table>
<thead>
<tr>
<th>data</th>
<th>RMSE</th>
<th>MAE</th>
<th>ACF1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenues forecast: pre-pandemic data</td>
<td>109151.6</td>
<td>79836.51</td>
<td>0.0252</td>
</tr>
<tr>
<td>Revenues forecast: post-pandemic data</td>
<td>183004.4</td>
<td>133228.9</td>
<td>0.0842</td>
</tr>
<tr>
<td>Number of orders: pre-pandemic data</td>
<td>17.8029</td>
<td>13.7436</td>
<td>0.0356</td>
</tr>
<tr>
<td>Number of orders: post-pandemic data</td>
<td>33.2794</td>
<td>25.0714</td>
<td>0.0460</td>
</tr>
</tbody>
</table>

Table 6. Accuracy results according to the *Theta*

<table>
<thead>
<tr>
<th>data</th>
<th>RMSE</th>
<th>MAE</th>
<th>ACF1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenues forecast: pre-pandemic data</td>
<td>123642.7</td>
<td>91222.12</td>
<td>0.0260</td>
</tr>
<tr>
<td>Revenues forecast: post-pandemic data</td>
<td>220112.3</td>
<td>158064.1</td>
<td>0.1135</td>
</tr>
<tr>
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<td>20.2868</td>
<td>15.8640</td>
<td>0.0650</td>
</tr>
<tr>
<td>Number of orders: post-pandemic data</td>
<td>37.4864</td>
<td>27.7061</td>
<td>0.1247</td>
</tr>
</tbody>
</table>

Table 7. Decline in model accuracy during the pandemic

<table>
<thead>
<tr>
<th></th>
<th>Theta</th>
<th>forecastHybrid</th>
<th>Nnar</th>
<th>ARIMA</th>
<th>ETS</th>
</tr>
</thead>
<tbody>
<tr>
<td>revenues</td>
<td>77.09%</td>
<td>70.07%</td>
<td>77.87%</td>
<td>91.77%</td>
<td>77.87%</td>
</tr>
<tr>
<td>number of orders</td>
<td>47.87%</td>
<td>22.61%</td>
<td>47.87%</td>
<td>74.07%</td>
<td>47.87%</td>
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</tbody>
</table>

Table 8. Order of models according to accuracy

<table>
<thead>
<tr>
<th>Order of models according to accuracy</th>
<th>RMSE</th>
<th>MAE</th>
<th>ACF1</th>
<th>Total order</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theta</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Revenues forecast: pre-pandemic data</td>
<td>2</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Revenues forecast: post-pandemic data</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>forecastHybrid</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Revenues forecast: pre-pandemic data</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Revenues forecast: post-pandemic data</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>NNAR</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Revenues forecast: pre-pandemic data</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Revenues forecast: post-pandemic data</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>ARIMA</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Revenues forecast: pre-pandemic data</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Revenues forecast: post-pandemic data</td>
<td>2</td>
<td>4</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>ETS</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Revenues forecast: pre-pandemic data</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Revenues forecast: post-pandemic data</td>
<td>3</td>
<td>3</td>
<td>3</td>
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</tr>
<tr>
<td>Theta</td>
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<tr>
<td>Number of orders: pre-pandemic data</td>
<td>3</td>
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<td>3</td>
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<tr>
<td>Number of orders: post-pandemic data</td>
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<td>3</td>
<td>3</td>
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<tr>
<td>forecastHybrid</td>
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<td></td>
</tr>
<tr>
<td>Number of orders: pre-pandemic data</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Number of orders: post-pandemic data</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>
Table 9. Continued

<table>
<thead>
<tr>
<th>Model</th>
<th>Number of orders: pre-pandemic data</th>
<th>Number of orders: post-pandemic data</th>
<th>Total order</th>
</tr>
</thead>
<tbody>
<tr>
<td>NNAR</td>
<td>RMSE: 3, MAE: 3, ACF1: 3</td>
<td>RMSE: 3, MAE: 3, ACF1: 3</td>
<td>3</td>
</tr>
<tr>
<td>ARIMA</td>
<td>RMSE: 2, MAE: 2, ACF1: 1</td>
<td>RMSE: 2, MAE: 2, ACF1: 1</td>
<td>2</td>
</tr>
<tr>
<td>ETS</td>
<td>RMSE: 3, MAE: 3, ACF1: 3</td>
<td>RMSE: 3, MAE: 3, ACF1: 3</td>
<td>3</td>
</tr>
</tbody>
</table>

Figure 1. Decomposition of additive time series daily revenues
Figure 2. Decomposition of additive time series daily number of orders

![Graph showing the decomposition of an additive time series into data, trend, seasonal, and remainder components.]

Source: own processing according to Hyndman and Athanasopoulos (2021).

Figure 3. Artificial neural networks

![Diagram of an artificial neural network with layers including synapses, bias, input signals, synaptic weights, and an activation function.]

Source: own processing according to Hyndman and Athanasopoulos (2021).
Figure 4. ETS Forecast

ETS(A,N,N): revenues forecast - pre-pandemic data

ETS(A,N,N): revenues forecast - post-pandemic data

ETS(A,N,N): number of orders forecast - pre-pandemic data

ETS(A,N,N): number of orders forecast - post-pandemic data
Figure 5. ARIMA forecast

ARIMA(0,1,1): revenues forecast – pre-pandemic data

ARIMA(0,1,2): revenues forecast – post-pandemic data

ARIMA(2,1,1): number of orders forecast – pre-pandemic data

ARIMA(0,1,2): number of orders forecast – post-pandemic data
Figure 6. Neural networks forecast

NNAR(14,8): revenues forecast – pre pandemic data

NNAR(26,14): revenues forecast – post-pandemic data

NNAR(14,8): number of orders forecast – pre-pandemic data

NNAR(14,8): number of orders forecast – post-pandemic data
Figure 7. hybridModel forecast

Hybrid Model: revenues forecast – pre-pandemic data

Hybrid Model: revenues forecast – post-pandemic data

Hybrid Model: number of orders forecast – pre-pandemic data

Hybrid Model: number of orders forecast – post-pandemic data
Figure 8. Theta forecast

Model Theta: revenues forecast - pre-pandemic data

Model Theta: revenues forecast - post-pandemic data

Model Theta: number of orders forecast - pre-pandemic data

Model Theta: number of orders forecast - post-pandemic data