Intra-market commonality in liquidity: new evidence from the Polish stock exchange

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Abstract
Research background: Empirical market microstructure research has recently shifted its focus from the examination of liquidity of individual securities towards analyses of the common determinants and components of liquidity. The identification of commonality in liquidity emerged as a new and fast growing strand of the literature on liquidity. However, the results around the world are ambiguous and rather depend on a specific stock market.

Purpose of the article: The aim of this study is to explore intra-market commonality in liquidity on the Warsaw Stock Exchange (WSE) by using daily proxies of six liquidity estimates: percentage relative spread, percentage realized spread, percentage price impact, percentage order ratio, modified turnover, and modified version of the Amihud measure. The sample covers a period from January 2005 to December 2016. The database contains the group of eighty-six WSE-listed companies.

Methods: The research hypothesis that there is commonality in liquidity on the Polish stock market is tested. The OLS with the HAC covariance matrix estimation and the GARCH-type models are employed to infer the patterns of liquidity co-movements on the WSE. Moreover, because the sample period is quite long, the stability of the empirical results by time period is examined. Seven 6-year time windows are utilized in the study.

Findings & Value added: The regression results reveal weak evidence of co-movements in liquidity on the WSE, regardless of the choice of the liquidity proxy. Furthermore, the robustness tests based on the time rolling-window approach do not unambiguously support the research hypothesis that there is commonality in liquidity on the Polish stock market. To the best of the author’s knowledge, the empirical findings presented here are novel and have not been reported in the literature thus far.
Introduction

Empirical market microstructure research has recently shifted its focus from the examination of liquidity of individual securities towards analyses of the common determinants and components of liquidity. The first empirical study of commonality in liquidity was conducted by Chordia et al. (2000). Using transactions data for the NYSE during 1992 and five measures of liquidity, the authors regressed individual stock daily percentage changes in liquidity on market and industry liquidity. Their results revealed that firm-level liquidity was significantly influenced by both a market and an industry-wide liquidity component.

Commonality in liquidity means that financial asset liquidity changes over time, and that these time variations are at least partly determined by a significant common component in the liquidity across assets. This phenomenon indicates that individual firm liquidity is sensitive to changes in aggregate liquidity. According to the literature, assessing commonality in liquidity is crucial for a number of reasons. The following topics are especially frequently explored and documented: the relationship between shareholders’ structure and individual firm liquidity, the consideration of commonality in liquidity in non-classical asset pricing models, the influence of commonality in liquidity on investment strategies, the importance of commonality in liquidity to central bankers and regulators, etc. Empirical evidence of common liquidity movements would assist regulators in improving market design (Narayan et al., 2015). Moreover, the existence of commonality in liquidity has important implications for asset pricing since it could represent a source of non-diversifiable risk (Olbryś, 2014).

However, the empirical findings concerning commonality in liquidity around the world are ambiguous and rather depend on a specific stock market. Therefore, the aim of this paper is to investigate intra-market commonality in liquidity on the Warsaw Stock Exchange (WSE). To the best of the author’s knowledge, the results on the WSE presented here are novel and have not been reported in the literature thus far.

We use six liquidity estimates for eighty-six WSE-traded companies in the period from January 2005 to December 2016. These liquidity measures are: (1) percentage relative spread, (2) percentage realized spread, (3) percentage price impact, (4) percentage order ratio, (5) the modified daily turnover, and (6) the modified version of daily Amihud (2002) illiquidity measure. Four liquidity proxies (i.e. percentage relative spread, percentage realized spread, percentage price impact, and percentage order ratio) are approximated using high frequency intraday data rounded to the nearest second. As the raw data set does not identify a trade direction on the WSE,
the Lee and Ready (1991) algorithm for classification of the initiator of a trade is employed to distinguish between so-called buyer- and seller-initiated trades. This information is essential for calculating the following liquidity proxies: percentage realized spread, percentage price impact, and percentage order ratio (Olbryś & Mursztyn, 2017). Furthermore, we use two liquidity measures that are approximated based on low frequency (daily) data. These proxies are: the modified daily turnover and the modified version of daily Amihud (2002) measure. All liquidity/illiquidity proxy time series utilized in this study have been previously assessed on the account of their various statistical properties and usefulness for commonality in liquidity investigation on the WSE (Olbrys & Mursztyn, 2018b).

This study tests the hypothesis that there is commonality in liquidity on the Polish stock market. In general, we utilize the research design of Chordia et al. (2000). However, we employ not only the linear regression with the HAC covariance matrix estimation (Newey & West, 1987), but also the GARCH-type models (if necessary) to infer the patterns of commonality in liquidity on the Polish stock market. Moreover, because the sample period is quite long (12 years), robustness analyses based on the 6-year rolling-window approach are provided.

The regression results reveal rather weak evidence of co-movements in liquidity on the WSE, regardless of the choice of the liquidity proxy. Furthermore, the empirical findings are robust to the choice of the 6-year time window. Therefore, no reason has been found to unambiguously support the hypothesis that there is commonality in liquidity on the WSE.

The remainder of the study is organized as follows. Section 2 contains a brief literature review regarding probable sources, implications, and empirical findings of liquidity co-movements around the world. Sections 3 and 4 present liquidity proxies used in the research. Section 5 describes the methodological background of assessing commonality in liquidity. Section 6 contains the empirical results for the WSE, as well as the robustness tests based on the time rolling-window approach. Section 7 discusses the nature and behavior of liquidity on the WSE. The last section summarizes the main results with conclusions and indicates further directions of the research.

**Literature review**

The literature concerning liquidity and commonality in liquidity is too vast to give a full citation list. As mentioned in Introduction, the existence of co-movements in liquidity suggests that there exists at least one common fac-
tor that simultaneously influences liquidity of all stocks in a market. From an investors’ point of view, the main question is whether they have to take liquidity risk into consideration in their financial decisions.

Therefore, there are some important aspects which justify tackling the problems presented in the paper. Firstly, we know relatively little about the fundamental sources that drive commonality in liquidity, and there is no unanimity in the literature regarding the causes of this phenomenon. For example, Karolyi et al. (2012) point out that one can distinguish between supply- and demand-side explanations for commonality in liquidity. The authors stress that some empirical studies have found support for supply-side sources of commonality in liquidity related to the funding constraints of financial intermediaries (Coughenour & Saad, 2004; Hameed et al., 2010). Other studies have explored demand-side sources driven by correlated trading activity (Chordia et al., 2000; Hasbrouck & Seppi, 2001), the level of institutional ownership (Kamara et al., 2008), and investor sentiment (Huberman & Halka, 2001).

Furthermore, findings on commonality in liquidity have raised a new issue of whether shocks in liquidity constitute a source of non-diversifiable risk. This is important because even if liquidity affects the risk of an asset, it should not be a priced risk factor if it is idiosyncratic and can be diversified away at the portfolio level. The literature has provided both theoretical and empirical evidence on the pricing of liquidity risk, but the results are ambiguous and rather depend on an individual stock market (e.g. Pastor & Stambaugh, 2003; Acharya & Pedersen, 2005; Bekaert et al., 2007; Korajczyk & Sadka, 2008; Martinez et al., 2005; Sadka, 2006; Watanabe & Watanabe, 2008; Lee, 2011; Olbryś, 2014; Foran et al., 2015; Ho & Chang, 2015; Stereniczak, 2019).

Beginning with Chordia et al. (2000), the identification of commonality in liquidity emerged as a new and fast growing strand of the literature on liquidity. Among others, Hasbrouck and Seppi (2001), Huberman and Halka (2001), Korajczyk and Sadka (2008), Kamara et al. (2008), and Kang and Zhang (2013) examined and documented common factors in liquidity, looking at the NYSE stocks. However, the results were ambiguous. While Hasbrouck and Seppi (2001) found strong evidence for common factors in order flows and stock returns but weaker evidence for commonality in liquidity proxies, Huberman and Halka (2001) obtained evidence suggesting the existence of a systematic liquidity component. More recent studies of commonality in the U.S. markets confirmed earlier results. In fact, Korajczyk and Sadka (2008) and Kamara et al. (2008) found evidence of commonality on the U.S. stock markets. Kang and Zhang (2013) examined the existence of limit order book commonality on the NYSE, and they
showed that inside liquidity provided by the limit order book exhibits much stronger commonality than outside liquidity.

There are also some empirical studies on commonality in liquidity for other individual equity markets in the world. Among others, Brockman and Chung (2002; 2006), Fabre and Frino (2004), Kempf and Mayston (2008), Pukthuanthong-Le and Visaltanachoti (2009), Foran et al. (2015), Narayan et al. (2015), and Miralles Marcelo et al. (2015) investigated the Stock Exchange of Hong Kong (SEHK), the Australian Stock Exchange (ASX), the Frankfurt Stock Exchange (FSE), the Stock Exchange in Thailand (SET), the London Stock Exchange (LSE), the Chinese stock exchanges (in Shanghai and Shenzhen), and the Euronext Lisbon Stock Exchange, respectively. It is pertinent to note that the empirical results on different stock markets in the world are ambiguous. For example, Fabre and Frino (2004) found no evidence to support commonality in liquidity on the ASX. On the other hand, Brockman and Chung (2002) reported the existence of commonality in liquidity on the SEHK. According to the literature, the inconsistent evidence of commonality in liquidity on the ASX and the SEHK might be attributed to the differences in market designs.

Moreover, some studies concern commonality in liquidity for the group of equity markets. Brockman et al. (2009) applied methodology of Chordia et al. (2000) to 47 stock exchanges. They documented the pervasive role of commonality in liquidity within individual exchanges. Karolyi et al. (2012) investigated cross-country commonality in liquidity based on daily data for individual stocks from 40 developed and emerging countries. Bai and Qin (2015) analyzed commonality in liquidity on 18 emerging markets. The authors pointed out that liquidity co-movements across emerging markets has a strong geographic component. The aforementioned three papers include Poland as one of emerging markets.

**Measuring liquidity from high frequency intraday data**

According to the literature, a quite extensive research on direct measurement of liquidity based on high frequency intraday data has been provided. Specifically, the literature indicates that different versions of a bid/ask spread are proper proxies for stock illiquidity because they estimate the cost of immediate execution of a trade. For example, the percentage relative spread (sometimes referred to as inside bid/ask spread or as proportional quoted spread) is commonly used as a measure for stock illiquidity (see e.g. Olbrys & Mursztyn, 2018a; 2018b and the references therein).
The bid/ask spread can be decomposed into permanent (informational) and transitory (immediacy-related) components (Glosten, 1987). Realized spread is a temporary component of the effective spread, which is described as the amount earned by a dealer or other supplier of immediacy (Huang & Stoll, 1996). Realized spread is sometimes referred to as a price reversal component, since a dealer takes profit only if price reverses. A proxy for price impact measures the sensitivity of a stock’s price to trades (Stoll, 2000), and most of researchers calculate price impact using intraday transaction data (see e.g. (Fong et al., 2017) and the references therein). Kyle (1985) introduces a theoretical model for such a measure based on the adverse information provided by a trade. Price impact could be defined as an increase (decrease) in the quote midpoint over a time interval beginning at the time of the buyer- (seller-) initiated trade. This is a permanent price change of a given transaction, or equivalently, a permanent component of the effective spread. Moreover, order imbalance has crucial influence on stock liquidity. Therefore, order imbalance indicators could be utilized among other liquidity and trading activity measures to approximate liquidity. The literature proposes various alternative proxies for order imbalance (see e.g. Chan & Fong, 2000; Chordia et al., 2002; Olbrys & Mursztyn, 2017; 2018a; Nowak, 2017).

In this study, four alternative estimates of liquidity/illiquidity derived from intraday data are utilized: (1) percentage relative spread, (2) percentage realized spread, (3) percentage proxy for price impact, and (4) the percentage order ratio as an indicator of order imbalance. To justify the measures selection for the WSE, it should be stressed that Olbrys and Mursztyn (2018a) propose five liquidity proxies including percentage effective spread, but their empirical findings unveil that daily values of percentage effective spread and percentage relative spread are almost the same for data from the WSE. Furthermore, percentage effective spread is factored into our research by its two components that complement each other: percentage realized spread and percentage price impact. Additionally, Olbrýś (2018a) tests stability of correlations between four liquidity proxies (excluding percentage effective spread) and her results confirm, that four liquidity estimates seem to capture various sources of market liquidity on the WSE.

**Percentage relative spread**

The database contains high frequency data rounded to the nearest second, i.e. opening, high, low and closing prices, as well as volume for a security over one unit of time.
The percentage relative spread value is given by Eq. (1):

\[
\%RS_t = \frac{100 \cdot (P_t^H - P_t^L)}{P_{t}^{mid}},
\]  

(1)

where \( P_t^H \), \( P_t^L \) are the highest and lowest prices at time \( t \), respectively, while the midpoint price \( P_{t}^{mid} \) at time \( t \) is given by the following Eq. (2):

\[
P_{t}^{mid} = \frac{P_t^H + P_t^L}{2}.
\]  

(2)

The midpoint price \( P_{t}^{mid} \) at time \( t \) is calculated as an arithmetic mean of the best ask price \( P_t(a) \) and the best bid price \( P_t(b) \) at time \( t \). Considering that the bid and ask prices are not made public on the WSE, the midpoint price at time \( t \) is rounded by an arithmetic mean of the lowest price \( P_t^L \) and the highest price \( P_t^H \) at time \( t \), which approximate the best ask price and the best bid price, respectively (Olbryś & Mursztyn, 2015).

Percentage relative spread (1) is in fact a measure of illiquidity. A high value of percentage relative spread denotes low liquidity. Conversely, a small value of this estimate denotes high liquidity. \( \%RS \) at time \( t \) is equal to zero when \( P_t^H = P_t^L \). The value of daily percentage relative spread is calculated as a volume-weighted average of percentage relative spreads computed over all the trades within a day (Olbrys & Mursztyn, 2018b).

**Three liquidity proxies supported by a trade side classification algorithm**

To compute some liquidity proxies using intraday data, it is crucial to recognize the side that initiates a transaction and to distinguish between so-called buyer- and seller-initiated trades. The WSE is an order-driven market with an electronic order book. However, information concerning the best bid and ask price is not publicly available. As a consequence, researchers should rely on indirect classification rules to infer the initiator of a trade. Various classification procedures of this type are described in the literature, but the Lee and Ready (1991) algorithm (LR) remains the most frequently used. For a brief literature review concerning trade classification rules see e.g. (Olbryś & Mursztyn, 2015; 2018c).
The LR algorithm operates in three steps (Theissen, 2001):

1. Transactions that occur at prices higher (lower) than the quote midpoint are classified as buyer-initiated (seller-initiated) trades.
2. Transactions that occur at a price that equals the quote midpoint but is higher (lower) than the previous transaction price are classified as being buyer-initiated (seller-initiated).
3. Transactions that occur at a price that equals both the quote midpoint and the previous transaction price but is higher (lower) than the last different transaction price are classified as being buyer-initiated (seller-initiated).

Moreover, the opening trade is treated as being unclassified. In this research, the LR algorithm is utilized as Olbrys and Mursztyn (2015; 2017; 2018c) confirm that this procedure performs quite well for data from the WSE. The empirical findings turn out to be robust to the choice of the sample and do not depend on firm’s size.

In this paper, three alternative proxies of liquidity/illiquidity derived from intraday data and supported by a trade side classification algorithm, are used: (1) percentage realized spread, (2) percentage price impact, and (3) the percentage order ratio as an indicator of order imbalance. Both the realized spread and price impact estimates are considered as the components of the effective spread, and they are computed over a time interval beginning at the moment of the buyer- or seller-initiated transaction. For example, Goyenko et al. (2009) and Fong et al. (2017) utilize a five-minute interval and the subscript \( t+5 \) defines trade five minutes after trade at time \( t \). Theissen (2001) proposes a more general approach and the subscript \( t+\tau \). In this study, the subscript \( t+5 \) indicates the fifth trade after the \( t \)-th trade (Olbryś & Mursztyn, 2017; 2018b).

1. Percentage realized spread

   Percentage realized spread is a temporary component of the effective spread and is given by Eq. (3):

   \[
   \%\text{Real}S_t = \begin{cases} 
   200 \cdot \ln \left( \frac{P_t}{P_{t+5}} \right), & \text{when trade } t \text{ is classified as buyer-initiated} \\
   200 \cdot \ln \left( \frac{P_{t+5}}{P_t} \right), & \text{when trade } t \text{ is classified as seller-initiated} 
   \end{cases},
   \tag{3}
   \]

   where the transaction price \( P_t \) at time \( t \) is approximated by the closing price. The price \( P_{t+5} \) is the closing price of the fifth trade after trade \( t \). \%\text{Real}S at moment \( t \) is equal to zero when \( P_t = P_{t+5} \). The value of daily
percentage realized spread is computed as a volume-weighted average of percentage realized spreads calculated over all the trades within a day. The value of daily percentage realized spread is defined to be equal to zero when all of the transactions within a day are unclassified (Olbrys & Mursztyn, 2018b).

### 2. Percentage price impact

Percentage price impact focuses on the change in the quote midpoint after a signed trade and is defined by Eq. (4):

\[
\%PI_t = \begin{cases} 
200 \cdot \ln \frac{p_{t+5}^{\text{mid}}}{p_t^{\text{mid}}} & \text{when trade } t \text{ is classified as buyer—initiated} \\
200 \cdot \ln \frac{p_t^{\text{mid}}}{p_{t+5}^{\text{mid}}} & \text{when trade } t \text{ is classified as seller—initiated}
\end{cases} 
\]

where the midpoint price \( p_t^{\text{mid}} \) at time \( t \) is given by Eq. (2), while \( p_{t+5}^{\text{mid}} \) is the quote midpoint of the fifth trade after trade \( t \). Price impact could be described as the increase (decrease) in the midpoint over a five trade interval beginning at the time of a buyer- (seller-) initiated transaction. \( \%\text{PI} \) at time \( t \) is equal to zero when \( p_t^{\text{mid}} = p_{t+5}^{\text{mid}} \). The proxy for daily percentage price impact is computed as a volume-weighted average of the estimates of percentage price impact calculated over all the trades within a day. The value of daily percentage price impact is defined to be equal to zero when all of the transactions within a day are unclassified (Olbrys & Mursztyn, 2018b).

### 3. Percentage order ratio

Percentage order ratio is utilized as an indicator of imbalance in daily orders and is given by Eq. (5):

\[
\%\text{OR} = 100 \cdot \frac{\left| \sum_{i=1}^{m} VBuy_i - \sum_{j=1}^{k} VSell_j \right|}{\sum_{n=1}^{N} V_n} 
\]

where the sums: \( \sum_{i=1}^{m} VBuy_i \), \( \sum_{j=1}^{k} VSell_j \), \( \sum_{n=1}^{N} V_n \) denote the daily cumulative volume of trading related to transactions classified as buyer- or seller-initiated trades, and daily cumulative volume of trading for all transactions, respectively. This indicator captures imbalance in the market since it rises as the difference in the numerator grows. A high value of the order ratio
denotes low liquidity. Conversely, a small value of the order ratio denotes high liquidity. The %OR indicator is equal to zero when the numerator is equal to zero. This happens when the daily cumulative volumes of trading related to transactions classified as buyer- and seller-initiated trades, respectively, are equal. The value of daily order ratio is defined to be equal to zero in the following two cases: (1) when all of the transactions within a day are unclassified, or (2) when the total volume of daily trading, in the denominator, is equal to zero (Olbrys & Mursztyn, 2018b).

**Measuring liquidity from low frequency daily data**

Direct measurement of liquidity is difficult and even impossible as intraday data are not freely available in the case of most emerging stock markets. A lack of access to high frequency data for emerging markets in general is a fact that is widely known and reported in the literature (e.g. Lesmond, 2005; Bekaert et al., 2007).

Given the uncertainty concerning liquidity estimation, some measures are especially often advocated in the literature to provide empirical study in liquidity/illiquidity effects in low frequency data. The popular measures of daily trading activity, i.e. volume, dollar trading volume, and share or market turnover are among them. Raw trading volume is the number of shares traded. The stock turnover is defined as the ratio of the number of shares traded in a day to the number of shares outstanding at the end of the day. It is worthwhile to note that using turnover disentangles the effect of a firm’s size from trading volume (Nowak & Olbryś, 2015). The market turnover is the ratio of the shares traded to market capitalization.

In this paper, two proxies of liquidity/illiquidity derived from daily data are calculated: (1) modified daily turnover and (2) modified version of the Amihud (2002) illiquidity proxy.

**Modified daily turnover**

A modified version of daily turnover, $MT_{i,d}$, as a measure of liquidity for stock $i$ on day $d$, is defined by Eq. (6):

$$MT_{i,d} = \log \left[ 1 + \frac{V_{i,d}}{NSO_{i,d}} \right] - \frac{1}{30} \cdot \sum_{k=1}^{30} \log \left[ 1 + \frac{V_{i,d-k}}{NSO_{i,d-k}} \right],$$  

(6)
where $V_{i,d}$ is the trading volume of stock $i$ on day $d$, and $NSO_{i,d}$ is the number of shares outstanding of stock $i$ on day $d$. Our method is based on Karolyi et al. (2012), but we use the number of shares outstanding at the beginning of the quarter for stock $i$ on day $d$ in equation (6), while Karolyi et al. use the number of shares outstanding at the beginning of the year. We calculate turnover in logs and de-trend the resulting series with a 30-day moving average to account for non-stationarity. The moving average is computed for the available data over the past 30 trading days. The empirical findings presented by Nowak and Olbryś (2015) unveil various day-of-the-week patterns in liquidity on the WSE. Therefore, it is important to note that using the modified version of daily turnover (6) disentangles these day-of-the-week effects from daily turnover (Olbrys & Mursztyn, 2018b).

**Modified version of the Amihud illiquidity proxy**

A modified version of the Amihud (2002) liquidity/illiquidity proxy, $MAmih_{i,d}$, is defined by Eq. (7):

$$MAmih_{i,d} = \begin{cases} \log \left(1 + \frac{|r_{i,d}|}{V_{i,d}}\right), & \text{when } V_{i,d} \neq 0, \\ 0, & \text{when } V_{i,d} = 0 \end{cases}$$

(7)

where $r_{i,d}$ is the simple rate of return of stock $i$ on day $d$, and $V_{i,d}$ is the trading volume of stock $i$ on day $d$. We follow Karolyi et al. (2012), but our method is slightly different, because the authors use return and volume in local currency, and finally multiply the result by negative one to obtain a variable that is increasing alongside with liquidity of individual stocks.

According to Eq. (7), the value of daily Amihud measure is defined to be equal to zero when the total volume of daily trading, in the denominator, is equal to zero. To avoid numerical problems, the daily values of the estimator (7) are rescaled by multiplying by $10^2$. In the literature, the Amihud measure is usually estimated monthly or for other periods (e.g. Goyenko et al., 2009; Olbryś, 2014; Vidović et al., 2014; Foran et al., 2015; Fong et al., 2017; Będowska-Sójka, 2018). In this paper, we calculate daily values of the Amihud proxy (Olbryś, 2018b).

**Research methodology**

To assess commonality in liquidity, the classical market model of liquidity, or the market and industry model of liquidity introduced by Chordía et al.
(2000) have been most frequently used. Moreover, various modifications of the models proposed by Chordia et al. (2000) have been presented in the literature (e.g. Coughenour & Saad, 2004; Brockman & Chung, 2006; Kempf & Mayston, 2008; Brockman et al., 2009; Pukthuanthong-Le & Visaltanachoti, 2009; Kang & Zhang, 2013; Foran et al., 2015; Miralles Marcelo et al., 2015; Bai & Qin, 2015).

In this study, we follow Olbryś (2018b) and employ the modified version of classical market model of liquidity of Chordia et al. (2000), including the Dimson’s (1979) correction for daily data (Eq. (8)):

\[
DL_{i,t} = \alpha_i + \beta_{i,-1} \cdot DL_{M,t-1} + \beta_{i,0} \cdot DL_{M,t} + \beta_{i,1} \cdot DL_{M,t+1} + \varepsilon_{i,t},
\]

(8)

where \(DL_{i,t}\) for stock \(i\) is the change in liquidity variable \(L\) from trading day \(t-1\) to \(t\), i.e. \(DL_t = \frac{L_t - L_{t-1}}{L_t}\). According to the Dimson’s procedure, the \(DL_{M,t-1}\), \(DL_{M,t}\), and \(DL_{M,t+1}\) are the lagged, contemporaneous (concurrent), and leading changes in a cross-sectional average of the liquidity variable \(L\), respectively. The Dimson’s correction enables us to accommodate the problem of nonsynchronous trading effects (Campbell et al., 1997).

It is important that in computing the ‘market’ liquidity proxy \(L_M\), stock \(i\) is excluded and the measure \(L_M\) is calculated as the equally-weighted average liquidity for the rest of stocks in the sample, hence the explanatory variables in model (8) are slightly different for each stock’s time series regression. Chordia et al. (2000) point out that changes are examined rather than levels because the interest is fundamentally in discovering whether liquidity moves. Based on model (8), positive and statistically significant slope coefficients are especially desired, as they indicate intra-market co-movements in liquidity and therefore confirm commonality in liquidity. In other words, they inform about liquidity co-movements in the same direction. A positive and significant coefficient would mean that exchange-level liquidity changes exert a substantial influence on a firm’s liquidity (Brockman et al., 2009).

For each stock, the model (8) is initially estimated by using the linear regression and the robust HAC estimates. However, the Newey and West (1987) method may not fully correct for the influence problems caused by the ARCH effect. Therefore, the estimation of the model (8) as a GARCH-type model is well-founded. To assess for the ARCH effect, the test of Engle (1982) with the Lagrange Multiplier (LM) statistic is used. In this research, the GARCH(p, q) model is utilized. According to the literature, the lower order GARCH(p, q), \(p, q = 1, 2\), models are employed in most
applications (Tsay, 2010). The GARCH(p, q) models are usually compared and selected by the Akaike (AIC) and Schwarz (SC) information criteria.

The GARCH(p, q) model is given by Eq. (9):

$$DL_{i,t} = a_i + \beta_{i,-1} \cdot DL_{M,t-1} + \beta_{i,0} \cdot DL_{M,t} + \beta_{i,+1} \cdot DL_{M,t+1} + \varepsilon_{i,t},$$

$$\varepsilon_{i,t} = z_{i,t} \sqrt{h_{i,t}}, \quad z_{i,t} \sim N(0,1),$$

$$h_{i,t} = a_{i,0} + \sum_{k=1}^{q} a_{i,k} \varepsilon_{i,t-k}^2 + \sum_{l=1}^{p} b_{i,l} h_{i,t-l},$$

where $a_{i,0} > 0$, $a_{i,k} \geq 0$, $k = 1, \ldots, q$, $q > 0$, $b_{i,l} \geq 0$, $l = \ldots, p$, $p \geq 0$.

Moreover, $\varepsilon_{i,t}$ is the innovation in a linear regression with $V(\varepsilon) = \sigma^2$, $h_{i,t}$ is the variance function, and remaining notation like in Eq. (8). The parameters of GARCH(p, q) models are almost invariably estimated via Maximum Likelihood (ML) or Quasi-Maximum Likelihood (QML) (Bollerslev & Wooldridge, 1992) methods, which bring up the subject of a suitable choice for the conditional distribution of innovation. Hamilton (2008) stresses that even if the researcher’s primary interest is in estimating the conditional mean, having a correct description of the conditional variance can still be quite important, because more efficient estimates of the conditional mean can be obtained in this case.

Results

Data description and results of commonality in liquidity on the WSE

In this research, two data samples are used (Olbrys & Mursztyn, 2018b). The first sample contains daily data (available at www.bossa.pl) for the group of eighty-six WSE-traded companies, in the period from January 2, 2005 to December 30, 2016 (3005 trading days). The quarterly number of shares outstanding of each stock is available at www.bankier.pl.

The second sample consists of high-frequency data rounded to the nearest second from the WSE (available at www.bossa.pl) for the same group of companies. The dataset contains the opening, high, low and closing prices, and volume for a security over one unit of time. The whole sample covers the same period from January 2, 2005 to December 30, 2016. In the database, only the securities that were traded on the WSE for the whole sample period since December 31, 2004 and were not suspended, were included. The 138 WSE companies met these basic conditions, and they
were initially selected. However, Nowak and Olbryś (2016) document that a large number of the WSE-listed companies unveil a substantial non-trading problem. To mitigate this problem, we excluded the stocks that exhibited extraordinarily many non-traded days during the whole sample period, precisely, above 300 zeros in daily volume, which constituted about 10% of all 3005 trading days. In this way, 104 companies were included in the database. In the next step, we excluded stocks that were suspended or removed from the WSE in 2017. Moreover, we perceived the problem of inconsistency between both intraday and daily data sets. We observed various gaps in data for some companies and hence we decided to exclude them from our database. Finally, 86 firms were entered into the database.

The foundation of time series analysis is stationarity (Tsay, 2010, p. 30). Therefore, we detected with the ADF-GLS test (Elliot et al., 1996) or ADF test (Dickey & Fuller, 1981) whether the analyzed daily time series are stationary. Using daily data, we utilize a maximum lag equal to five and then remove lags until the last one is statistically significant (Adkins, 2014). The critical values of the ADF-GLS or ADF \( \tau \)-statistics for the rejection of the null hypothesis of a unit root are presented in Elliott et al. (1996), Cook and Manning (2004), MacKinnon (2010). We proved that the unit-root hypothesis can be rejected at the 5 per cent significance level for all-time series utilized in the study.\(^1\)

In order to reduce the effects of possibly spurious outliers, we ‘winso-rized’ the data by using the 1st and 99th percentiles for each time series (e.g. Korajczyk & Sadka, 2008; Kamara et al., 2008).

In the next step, we employed the linear regression with the HAC covariance matrix estimator to calculate the parameters of the model (8). In total, 516 models were estimated. For each stock, daily proportional changes in individual stock liquidity variables were regressed in time-series on the changes of an equally weighted cross-sectional average of the liquidity variable for all stocks in the sample, excluding the dependent variable stock. In the case of 93 models (comprising 18 models for %RS, 22 models for %RealS, 21 models for %PI, 4 models for %OR, 8 models for MT, and 20 models for MAmih measure), the ARCH effect in residuals was detected. Therefore, for those 93 companies and liquidity proxies the GARCH(p, q), \( p, q = 1, 2 \), models were estimated. The number of lags \( p, q \) was selected on the basis of the Akaike (AIC) and Schwarz (SC) information criteria.\(^2\)

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\(^1\) Due to the space restriction, details are available upon a request, because the number of time series is large, i.e. there are 1032 = 6 \cdot 86 + 6 \cdot 86 daily time series in total.

\(^2\) The SUR (Seemingly Unrelated Regression) method, proposed by Zellner (1962), has also been applied, because it seemed to be appropriate for our panel data. The Breusch-Pagan (1980) Lagrange Multiplier statistic has been used to test for the existence of contem-
The summarized cross-sectional estimation results of models (8) — (9) are presented in Table 1. This table contains the proportion of positive, positive significant, negative, and negative significant coefficients.

The overall cross-sectional findings presented in Table 1 need comments. There is some evidence of intra-market co-movements in liquidity, but it is definitely less significant and less pervasive than that presented by Chordia et al. (2000) in their seminal paper. Our results are rather similar to those reported by Fabre and Frino (2004) for the Australian Stock Exchange, which is a pure order-driven market like the WSE. The regressions provide weak evidence of commonality in liquidity on the WSE, regardless of the choice of the liquidity measure, because positive and statistically significant coefficients are scarce. For example, the positive and statistically significant concurrent coefficients constitute only: 13.95%, 8.14%, 3.49%, 6.98%, 8.14%, and 8.14% of all concurrent coefficients for D%RS, D%RealS, D%PI, D%OR, DMT, and DMAmih models, respectively. The results for different liquidity proxies are slightly diverse, but they are quantitatively similar.

Robustness analyses

The stability of the empirical results by time period is examined. The whole sample period is quite long (12 years), therefore robustness tests based on the 6-year rolling-window approach are provided. We utilize seven 6-year time windows: (1) Window 1 (5.01.2005–31.12.2010), (2) Window 2 (2.01.2006–30.12.2011), (3) Window 3 (2.01.2007–28.12.2012), (4) Window 4 (2.01.2008–30.12.2013), (5) Window 5 (5.01.2009–30.12.2014), (6) Window 6 (4.01.2010–30.12.2015), and (7) Window 7 (3.01.2011–29.12.2016). We estimate the parameters of the model (8) for each stock and liquidity proxy, within each window. A large number of models (3612 = 7 · 516) has been investigated. Summarized time rolling-window results are presented in Table 2.

The summarized results of rolling-window analyses reported in Table 2 reveal that positive and statistically significant coefficients appear rarely, regardless of the choice of the liquidity estimate. Moreover, the number of positive and negative statistically significant coefficients is similar for almost all liquidity proxies, except for the modified Amihud measure. In the case of the MAmih estimate (Eq. 7), the proportion of negative and statisti-
cally significant coefficients for each window is even greater than the proportion of positive and statistically significant coefficients. This evidence informs about liquidity movement in the opposite direction, but it does not concern the whole sample period (see Table 1). In our opinion, this phenomenon observed for the MAmih estimate could be explained by its relatively high sensitivity to nonsynchronous trading effects (see the next section), as this measure is calculated based on daily rate of return.

To sum up, the stability tests based on the 6-year rolling-window approach do not unambiguously support the research hypothesis that there is commonality in liquidity on the Polish stock market.

Discussion

The Warsaw Stock Exchange is large compared to the other Central and Eastern European stock exchanges. For comparison, at the end of 2016 the total number of listed stocks was equal to: 881 (Warsaw), 23 (Prague), 41 (Budapest), 71 (Bratislava), 37 (Ljubljana), 34 (Vilnius), 17 (Tallinn), and 32 (Riga) (Olbryś, 2018b). One could expect that the WSE is a liquid market. Unfortunately, a large number of the WSE-traded companies reveal a substantial non-trading problem, which means a lack of transactions over a particular period when the stock exchange is open for trading. This phenomenon may be considered as a special case of the nonsynchronous trading effect. Nowak and Olbryś (2016) documented that the average amount of non-traded days is not significantly larger for smaller firms, so the non-trading problem does not depend on a firm’s size. The non-trading effect is usually placed in a broad class of market frictions. In the literature, frictions are understood as various disturbances in trading processes, and they have some important theoretical and empirical implications. Among others, the presence of frictions causes market illiquidity, and therefore it plays a significant role in asset pricing on the WSE (e.g. Olbryś, 2014; Stereńczak, 2019). Moreover, it is well known fact that the non-trading effect induces potentially serious biases in various statistical measures of asset returns (see e.g. (Nowak & Olbryś, 2016) and the references therein). Another strand of the literature concerns price jumps, which may be treated as market frictions. Będowska-Sójka (2016) analyzed the behavior of liquidity measures around the time of price jumps on the WSE. She documented that jumps are accompanied by abnormally high increases in some liquidity proxies, hence jumps occur together with the substantial liquidity pressure. Aforementioned reasons help us to understand that a rather weak evidence of commonality in liquidity on the WSE is not very surprising.
Conclusions

Commonality in liquidity is nowadays the center of attention of many empirical studies. Therefore, the main goal of this paper was to explore and document commonality in liquidity patterns on the WSE, using six alternative liquidity proxies based on intraday or daily data for a broad sample of stocks. To address this issue, the OLS-HAC estimation and the GARCH-type models have been employed. The research has provided evidence for statistically insignificant intra-market co-movements in liquidity, regardless of the choice of the liquidity measure. Therefore, no reason has been found to support commonality in liquidity on the WSE, i.e. liquidity rather does not co-move on the Polish stock exchange. As would be anticipated, the empirical results are consistent with the existing literature concerning the Polish stock market. For example, Będowska-Sójka (2019) employed different liquidity measures, but she also found that commonality in liquidity on the WSE is weak and robust to the choice of liquidity proxy. Moreover, it is important that these findings are in accordance with the investor’s intuition because, as mentioned in the previous section, a large number of the WSE-traded companies reveal a substantial non-trading problem.

On the contrary, the evidence of no commonality in liquidity is rather in contrast to previous studies for the U.S. developed market. However, apart from a market size, the WSE is a pure order-driven stock market with an electronic order book, and it differs from the NYSE and the NASDAQ. The U.S. stock exchanges are hybrid markets. Therefore, the empirical results obtained for the U.S. stock market are not comparable to the Polish stock market in many aspects. In general, the probable explanation of discrepancies in liquidity/illiquidity between markets is that stock market structure and trading mechanisms may affect the level of liquidity.

Given the importance of the topic, one of possible directions for further investigation could be to identify the components of liquidity on the WSE by using methods based on the principal component approach. The PCA method has been applied by Hasbrouck and Seppi (2001) and Chen (2005) among others. The authors employed principal component analysis for constructing factors to maximize explanatory power within a set of related variables. The main goal was to extract a common source of liquidity variation. The APC procedure has been used e.g. by Korajczyk and Sadka (2008), and Foran et al. (2015). In this procedure, the asymptotic principal component analysis is utilized to capture systematic variations or commonality in liquidity across stocks. To the best of the author’s knowledge, no such research has been so far undertaken for the Polish stock market.
References


Acknowledgments

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### Annex

**Table 1.** Testing for market-wide commonality in liquidity on the Warsaw Stock Exchange in the whole sample period from January 2, 2005 to December 30, 2016 (3005 trading days)

<table>
<thead>
<tr>
<th></th>
<th>D%RS</th>
<th>D%RealS</th>
<th>D%PI</th>
</tr>
</thead>
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<tr>
<td></td>
<td>OLS-HAC</td>
<td>GARCH</td>
<td>OLS-HAC</td>
</tr>
<tr>
<td></td>
<td>68 models</td>
<td>Conditional mean equation</td>
<td>18 models</td>
</tr>
<tr>
<td>Concurrent $\beta_{t,0}$</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>% positive</td>
<td>58.83</td>
<td>72.22</td>
<td>53.12</td>
</tr>
<tr>
<td>% positive significant</td>
<td>10.29</td>
<td>27.78</td>
<td>7.81</td>
</tr>
<tr>
<td>% negative</td>
<td>41.17</td>
<td>27.78</td>
<td>48.88</td>
</tr>
<tr>
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<td>7.35</td>
<td>0</td>
<td>6.25</td>
</tr>
<tr>
<td>% positive significant (all 86 models)</td>
<td>13.95</td>
<td>8.14</td>
<td>3.49</td>
</tr>
<tr>
<td>Lag $\beta_{t-1}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% positive</td>
<td>44.12</td>
<td>38.89</td>
<td>60.93</td>
</tr>
<tr>
<td>% positive significant</td>
<td>4.41</td>
<td>11.11</td>
<td>6.25</td>
</tr>
<tr>
<td>% negative</td>
<td>55.88</td>
<td>61.11</td>
<td>39.07</td>
</tr>
<tr>
<td>% negative significant</td>
<td>4.41</td>
<td>11.11</td>
<td>4.69</td>
</tr>
<tr>
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</tr>
<tr>
<td>Lead $\beta_{t+1}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% positive</td>
<td>51.47</td>
<td>38.86</td>
<td>56.25</td>
</tr>
<tr>
<td>% positive significant</td>
<td>8.82</td>
<td>5.56</td>
<td>4.69</td>
</tr>
<tr>
<td>% negative</td>
<td>48.53</td>
<td>61.14</td>
<td>43.75</td>
</tr>
<tr>
<td>% negative significant</td>
<td>5.88</td>
<td>0</td>
<td>6.25</td>
</tr>
<tr>
<td>% positive significant (all 86 models)</td>
<td>8.14</td>
<td>4.65</td>
<td>8.14</td>
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Table 1. Continued

<table>
<thead>
<tr>
<th></th>
<th>D% OR</th>
<th>GARCH</th>
<th>DMT</th>
<th>GARCH</th>
<th>DMAmih</th>
</tr>
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<tr>
<td></td>
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<td>Conditional mean equation</td>
<td>OLS-HAC</td>
<td>Conditional mean equation</td>
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<td>Concurrent $\beta_{t,0}$</td>
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<td>51.28</td>
<td>12.5</td>
<td>51.51</td>
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<td>% positive</td>
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<td>8.97</td>
<td>0</td>
<td>9.09</td>
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<td>% positive significant</td>
<td>39.02</td>
<td>50</td>
<td>48.72</td>
<td>87.5</td>
<td>48.49</td>
</tr>
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<td>% negative</td>
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<td>0</td>
<td>6.06</td>
</tr>
<tr>
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<td>8.14</td>
<td>8.14</td>
<td>1.16</td>
<td>10.47</td>
</tr>
</tbody>
</table>

| Lag $\beta_{t-1}$ | 40.24  | 75   | 62.82 | 62.5  | 48.48  | 40  |
| % positive       | 1.22   | 0    | 11.54 | 0     | 3.03   | 5   |
| % positive significant | 59.76 | 25   | 37.18 | 37.5  | 51.52  | 60  |
| % negative       | 4.88   | 0    | 2.56  | 12.5  | 7.58   | 15  |
| % negative significant | 1.16 | 10.47 | 3.49  | 2.33  | 3.49  |

| Lead $\beta_{t+1}$ | 50    | 75   | 38.46 | 50    | 56.07  | 45  |
| % positive       | 2.44  | 25   | 2.56  | 0     | 4.55   | 0   |
| % positive significant | 50   | 25   | 61.54 | 50    | 43.93  | 55  |
| % negative       | 6.10  | 0    | 3.85  | 0     | 9.09   | 15  |
| % negative significant | 3.49 | 2.33 | 3.49  | 3.49  |

Notes: %RS is the percentage relative spread (Eq. 2). %RealS is the percentage realized spread (Eq. 3). %PI is the percentage price impact (Eq. 4). %OR is the percentage order ratio (Eq. 5). MT is the modified turnover (Eq. 6). MAmih is the modified Amihud measure (Eq. 7). 'D' preceding the acronym denotes a proportional change in the variable across successive trading days. The most important percentage of positive and significant coefficients is marked in bold.

% positive – the percentage of positive slope coefficients. % positive significant – the percentage of positive and significant slope coefficients. % negative – the percentage of negative slope coefficients. % negative significant – the percentage of negative and significant slope coefficients.
Table 2. The rolling-window results of testing for market-wide commonality in liquidity on the Warsaw Stock Exchange

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>The proportion of positive /negative and statistically significant slope coefficients (the total number of estimated models is equal to 86 for each window)</th>
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<tr>
<td></td>
<td>Window 1</td>
</tr>
<tr>
<td>D%RS</td>
<td></td>
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<td></td>
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<tr>
<td>$\beta_{t,0}$</td>
<td>12/6</td>
</tr>
<tr>
<td>Lag $\beta_{t-1}$</td>
<td>7/6</td>
</tr>
<tr>
<td>Lead $\beta_{t+1}$</td>
<td>9/7</td>
</tr>
<tr>
<td>D%RealS</td>
<td></td>
</tr>
<tr>
<td>Concurrent</td>
<td></td>
</tr>
<tr>
<td>$\beta_{t,0}$</td>
<td>4/5</td>
</tr>
<tr>
<td>Lag $\beta_{t-1}$</td>
<td>7/1</td>
</tr>
<tr>
<td>Lead $\beta_{t+1}$</td>
<td>6/4</td>
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<tr>
<td>D%PI</td>
<td></td>
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<tr>
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<td>$\beta_{t,0}$</td>
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</tr>
<tr>
<td>Lag $\beta_{t-1}$</td>
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