Gawrońska-Nowak, B., & Grabowski, W. (2016). Using Genetic Algorithm in Dynamic Model of Speculative Attack. *Equilibrium. Quarterly Journal of Economics and Economic Policy*, 11(2), 287-306. DOI: <http://dx.doi.org/10.12775/EQUIL.2016.013>

Bogna Gawrońska-Nowak*

Lazarski University, Poland

Wojciech Grabowski

University of Lodz, Poland

Using Genetic Algorithm in Dynamic Model of Speculative Attack

JEL Classification: C6; F3; E5

Keywords: *currency crisis; dynamic model; genetic algorithms*

Abstract: *Evolution of speculative attack models shows certain progress in developing the idea of the role of expectations in the crisis mechanism. Obstfeld (1996) defines expectations as fully exogenous. Morris and Shin (1998) treat the expectations as endogenous (with respect to noise), not devoting too much attention to information structure of the foreign exchange market. Dynamic approach proposed by Angeletos, Hellwig and Pavan (2006) offers more sophisticated assumption about learning process. It tries to reflect time-variant and complex nature of information. However, this model ignores many important details like a Central Bank cost function. Genetic algorithm allows to avoid problems connected with incorporating information and expectations into agent decision-making process to an extent. There are some similarities between the evolution in Nature and currency market performance. In our paper an assumption about rational agent behaviour in the efficient market is criticised and we present our version of the dynamic model of a speculative attack, in which we use a genetic algorithm (GA) to define decision-making process of the currency market agents. The results of our simula-*

tion seem to be in line with the theory and intuition. An advantage of our model is that it reflects reality in a quite complex way, i.e. level of noise changes in time (decreasing), there are different states of fundamentals (with “more sensitive” upper part of the scale), the number of inflowing agents can be low or high (due to different globalization phases, different capital flow phases, different uncertainty levels).

Introduction

Speculative attack models try to catch a complicated relation between information and expectations. Informed agents (provided by either private signals or common knowledge or both of these) formulate their expectations, and due to these expectations make strategies either to attack or hold. This mechanism from information through expectations to attack has always been extremely difficult to cover in any theoretical framework. Evolution of speculative attack models show certain progress in developing idea of the role of expectations in the crisis mechanism. Obstfeld (1996) defines expectations as fully exogenous. Morris and Shin (1998) treat the expectations as endogenous (with respect to noise), not devoting too much attention to the information structure of the foreign exchange market. They propose a static model, including information but excluding any possibility of so-called common knowledge in the currency market. Dynamic approach proposed by Angeletos, Hellwig and Pavan (2006) offers more sophisticated assumption about the learning process. It tries to reflect time-variant and complex nature of information. However, the model ignores many important details like for example a Central Bank cost function.

If we look at the speculative attack as at the optimisation problem, why not to use genetic algorithm to present the agent behaviour in the market? Genetic algorithm allows to avoid problems connected with incorporating information and expectations into agent decision making process to an extent. Evolution means that the species that are worse prepared to the environment, have smaller chances to survive, and as the time passes by, improved species appear. There are some similarities between the evolution in Nature and currency market performance. In the currency market, the speculators make wrong decisions and are eliminated from the market by these speculators who generate high pay-offs. Therefore, we can assume that learning in the currency market may in fact be characterised like species adaptation process to the environment. That is why we believe that introducing genetic algorithm may be a right step towards finding some optimal solutions for the speculative attack model.

This paper is organized as follows. In the first section, the assumption concerning rational agent behaviour in the efficient market is criticised, and we explain why we use genetic algorithm. In the second section, dynamic model of a speculative attack is presented. In the third section, optimal strategies for the Central Bank and for speculators are defined. In the fourth section, genetic algorithm that reflects decision making process is described. In the fifth section, our results are presented. We also show the evolution of the learning process. The last section contains conclusions.

Some Critical Points Towards Agent Behaviour in the Foreign Exchange Market

Foreign exchange market cannot be characterised as a good example of the strong efficiency paradigm by Fama (1970). Information is not equally available to all agents. The market is rather decentralised and trade transparency is low (see: Lyons, 2001). It is well known that this makes the foreign exchange market special among other financial markets.

Using behavioural finance perspective, we can say that although an agent may store and process only a tiny part of the relevant information, the agent is not brainless. If we agree to abandon traditional rational expectation model that assumes perfect knowledge of the market participants, then it is possible to redefine an individual forecasting strategy, which is neither fully rational (in a sense of *homo oeconomicus*) nor fully irrational. It is in line with heuristics rules taken from psychology. So-called *trial and error* strategy represents bounded rationality framework and means *ex post* checking how profitable a certain rule is, while comparing it with some others. If the rule does not prove to be a profitable one, then the agent switches to a better one. If the agent's strategy turns out to be successful, then she/he sticks to it. *Trial and error* strategy is rooted in Nature and has got a strongly evolutionary character.

In the behavioural model of exchange rate by De Grauwe and Grimaldi (2006) the mechanism of making forecasts by the agents is well described. The authors show that in the foreign exchange market the agents follow *trial and error* strategy, no matter if they are so-called "fundamentalists" or "chartists" (no matter if they analyse macroeconomic fundamentals or they rely on technical analysis to forecast the exchange rate). *Ex post* assessment of the forecasting strategies may transform "fundamentalist" into "chartist" or vice versa. It is worth mentioning that, according to Tversky, Kahneman (1991), the agents need some time to adopt a new strategy, they are slightly conservative, therefore "status quo bias" must be considered in their deci-

sion making process, even though it is true that the agents react to the relative profitability of the rules. *Trial and error* strategy is thus a dynamic process that requires further assumptions concerning “memory” of the agent. De Grauwe and Grimaldi (2006) use the short-run memory hypothesis that implies that the agents refer just to last period’s squared forecast error to make their decision.

Frydman and Goldberg (2007) formulate some critical remarks towards the rational expectation and efficient market hypothesis as well. They are quite close to the behavioural economists’ point of view. The authors pay attention to the fact that the individuals in the foreign exchange market must cope with imperfect knowledge. They stress the importance of the revision of the agent forecasting strategies over time at the same time mentioning that even “social context” should be considered as an important determinant of the strategy formulation process. They also describe the agents as conservative, defining this as follows: “*an individual’s forecast of the future exchange rate is not too different from the forecast she would have had if she did not revise her forecasting strategy*” (Frydman & Goldberg, 2007, p. 184).

It seems that formulating a model that would reflect true agent behaviour in the foreign exchange market in a proper way is more complicated task than the supporters of traditional efficient market hypothesis would like to present. Such a model should have an evolutionary, dynamic character, show making decision processes based on *trial and error* strategy which are treated as optimisation, however, under imperfect knowledge assumption. Genetic algorithm appears to be quite suitable to imitate agents’ behaviour in the foreign exchange market in the real world if we want to meet majority of these criteria.

Neural networks and genetic algorithms have proved to be quite efficient methods used to analyse financial (including currency) crisis episodes (Aydin & Cavdar, 2015; Sarlin, 2014; Arifovic & Maschek, 2012). Applying evolutionary algorithms allows to avoid suffering from some of the typical problems connected with working with financial data, such as volatility clustering or fat tails of the time series. Moreover, simulations based on the genetic algorithms have relatively high explanatory power and they quite often outperform traditional model outputs. They rely on a so-called Social Learning (SL) idea where a population of beliefs of a large number of agents evolves together over time. This concept reflects well the fact that a large number of investors participating in trading observe and imitate the behaviour of some of the other investors. From time to time they may also try to adapt new, different rules, which is to be done experimentally (very much in line with the *trial and error* strategy, mentioned before). All of

these assumptions seem to play an important role in capturing the distinguishing features of the foreign exchange market, and therefore influence significantly the final model quality. In our opinion, GA usage advantages should not be ignored, especially, if we keep in mind that there is a very limited access to field data, and in consequence the speculative attack models have always been very theoretical.

Dynamic Model of Speculative Attack

Both models by Obstfeld (1986) and Morris and Shin (1998) have some shortcomings. In our paper, we try to develop these models and make them more applicable. Neither “multiple equilibria” approach nor “uniqueness” take into account time as an important factor, as they are both static. Therefore, in our paper dynamics of the model is introduced. We adapt some of the model elements proposed by Angelestos *et al.* (2006). Their model offers a rather general framework of how to apply dynamic global games into a regime change mechanism. It can be applied for modelling speculation against a currency peg (which is of our special interest), at the same time the model can be also used for some other purposes, like explaining run against a bank or some other (not strictly economic) processes, for example a revolution against a dictator. There are two important features of the model. Firstly, it allows the agents to learn. Secondly, the fundamentals matter for the regime outcome prediction, although not for timing and number of attacks. At the same time, some drawbacks of the model should be mentioned. The model presents only one-side perspective, i.e. the speculator one, and the payoff function of the Central Bank is not analysed. Moreover, we are not quite sure if it is fully satisfying to accept:

*“summarizing the private information by the agent about θ at any given period in a one dimensional sufficient statistic, and capturing the dynamics of the cross-sectional distribution of the static in a parsimonious way (Angelestos *et al.*, 2006, pp. 1-2)”*,

and then to apply this algorithm to examine the effects of learning on equilibria in the model.

Instead, we offer well defined genetic algorithm to simulate learning process, and as we think that a Central Bank can also learn, in fact the genetic algorithm is used to show how decisions of two categories of agents are changing while their knowledge on the proportion of attacking specula-

tors is being changed too. Below, we present the main assumptions of our dynamic model:

- 1) Time is discrete and indexed by $t \in \{1, 2, \dots\}$. Agents are indexed by $n \in \{1, \dots, N_t, N_t + 1\}$, where agents $1, \dots, N_t$ are speculators and agent $N_t + 1$ is the Central Bank. Subscript t is used, since we assume that number of speculators considering attack evaluates in time. Therefore, there is a sequence $\{N_t\}_{t \in \{1, 2, \dots\}}$, which is not observed by agents. Since financial markets develop and number of institutional investors increases, it is expected that the number of speculators considering attack increases in time. In fact, speculators with the worst results are replaced by new speculators, however we assume that the number of speculators is an increasing function of time. We assume that the total number of speculators evolves according to the following formula:

$$N_t = N_1 + (t - 1)\tau, \quad t = 1, 2, \dots \quad (1)$$

It should be stressed that the agents do not know the value of parameter τ . They predict the parameter value.

- 2) The Central Bank receives ex post information about the number of speculators attacking. We assume that speculators attack with the probability:

$$PA_t^n = \kappa_t b^n, t = 1, 2, \dots \quad n = 1, 2, \dots, N_t, \quad (2)$$

where κ_t evolves in time and depends on the state of macroeconomic fundamentals and (predicted by speculators) probability of abandoning the exchange rate peg. b^n does not change in time and reflects heterogeneity of speculators. We assume that for each n , $b^n \sim U(0, 1)$. The larger is value of the parameter b^n , the higher is propensity of a speculator to attack. The lower is value of this parameter the more risk averse is speculator. In addition we assume that $E(PA_t^n) = E(\kappa_t)E(b^n) = 0.5E(\kappa_t)$.

- 3) We define the sequence of observed exchange rates $\{er_t\}_{t \in \{1, 2, \dots\}}$ and the sequence of true values of macroeconomic fundamentals $\{\theta_t\}_{t \in \{1, 2, \dots\}}$. We assume that the Central Bank is interested in pegging the exchange

rate. Therefore, similarly as in the model of Morris and Shin (1998), there are only two possible states of the exchange rate. Exchange rate is pegged at a level e^* or depends on the fundamentals (it means it is floating) and equals $f(\theta)$. An action set for the Central Bank is binary, which means that the Central Bank can either defend the exchange rate peg or abandon it. Since speculators can attack the exchange rate peg or refrain from doing so, their action set is binary as well. We assume that in the first period the exchange rate is pegged, so $er_1 = e^*$. The game is continued until a state $er_t = f(\theta_t)$ is reached or if after a finite number of periods dominant strategy is not to attack.

- 4) According to the model of Angelestsos *et al.* (2006) each player receives a private signal $x_t = \theta_t + \varepsilon_t^n$, where $\varepsilon_t^n \sim N\left(0, \frac{1}{\beta_t}\right)$ is a noise for $n = 1, 2, \dots, N_t$. This noise is independent and identically distributed across agents. In the case of the Central Bank, we assume that a noise $\varepsilon_t^{N_t+1} \sim N\left(0, \frac{1}{\tilde{\beta}_t}\right)$ is independent of noises $\varepsilon_t^1, \dots, \varepsilon_t^{N_t}$. In addition we assume that the knowledge about the level of fundamentals is more precise in the case of the Central Bank than in a case of speculators, which means that $\tilde{\beta}_t > \beta_t$. It is also assumed that uncertainty concerning the level of fundamentals decreases in time, and therefore: $\forall_{s>t} \beta_s > \beta_t$.
- 5) In this paper $c(\cdot)$ denotes the Central Bank cost function associated with defending the exchange rate peg. This cost depends on the state of fundamentals, total number of speculators considering attacking and the probability that a chosen speculator attacks and the level of reserves. Since the total number of speculators considering attacking depends on the initial number of speculators and average increase of the number of speculators, we finally have:

$$c_t = c_t(N_1, \tau, PA_t^n, \theta_t, r_t). \quad (3)$$

We assume that the function (3) is continuous and

$$\frac{\partial c_t(N_1, \tau, PA_t^n, \theta_t, r_t)}{\partial N_1} > 0, \quad \frac{\partial c_t(N_1, \tau, PA_t^n, \theta_t, r_t)}{\partial \tau} > 0,$$

$$\frac{\partial c_t(N_1, \tau, PA_t^n, \theta_t, r_t)}{\partial PA_t^n} > 0, \quad \frac{\partial c_t(N_1, \tau, PA_t^n, \theta_t, r_t)}{\partial \theta_t} < 0 \quad \text{and}$$

$$\frac{\partial c_t(N_1, \tau, PA_t^n, \theta_t, r_t)}{\partial r_t} < 0. \text{ These assumptions seem to be realistic, since:}$$

- The more reserves the Central Bank possesses, the greater its possibilities of defending the exchange rate peg are.
- If the state of macroeconomic fundamentals is good, then government does not have to conduct active fiscal policy and there is no pressure on the Central Bank to act and its independence is not questioned. Therefore, the Central Bank can concentrate rather on defending the currency peg.
- The bigger number of speculators considering attacking is, the harder defending currency peg is.
- Our next necessary step to develop previous models is to specify the cost function. We decide to use a linear specification:

$$c_t = \gamma_1 \sum_{n=1}^{N_1 + \tau(t-2)} PA_{t-1}^n + \gamma_2 \theta_{t-1} + \gamma_3 r_{t-1}, \quad t = 2, 3, \dots \quad (4)$$

- 6) All agents do action in the period $t-1$ and the result of this action is observed in period t . Let $\{ST_t^n\}_{t \in \{1, 2, \dots\}}$ denote a sequence of strategies chosen by the n -th agent, while $pay_t^n(ST_{t-1}^n)$ denotes the payoff in period t for an agent number n , if this agent chooses action ST_{t-1}^n in period $t-1$. We assume that the speculators either decide to attack currency ($ST_t^n = 1$) or refrain from doing so ($ST_t^n = 0$). In the case of the Central Bank, it defends the exchange rate peg ($ST_t^{N_t+1} = 1$) or abandons it ($ST_t^{N_t+1} = 0$). Therefore for each n $ST_t^n \in \{0, 1\}$. If the Central Bank succeeds, it receives payoff v . The attacking speculators bear the transaction cost τr . They succeed if the Central Bank abandons the exchange rate peg and they receive payoff $e^* - f(\theta_t)$, which depends on the scale of devaluation.

Optimal Strategies for the Central Bank and for Speculators

The payoff of speculator depends on his/her decision and on the decision made by the Central Bank. Since there are two possible strategies of the Central Bank and two possible strategies for each speculator, we have finally the following 2X2 matrix with payoffs of speculators.

Table 1. Payoffs for speculators in period t ($er_{t-1} = e^*$)

$ST_{t-1}^n \backslash ST_{t-1}^{N_{t-1}+1}$	0	1
0	0	0
1	$(e^* - f(\theta_t)) - tr$	$-tr$

Source: own work.

The payoff of the Central Bank depends on the proportion of speculators attacking, state of fundamentals and the level of reserves. The payoff in period t conditional on the action made in period $t-1$ is defined in the following way:

$$pay_t^{N_t+1}(ST_{t-1}^{N_{t-1}+1}) = \begin{cases} 0 & \text{if } ST_{t-1}^{N_{t-1}+1} = 0, \\ v - \gamma_1 \sum_{n=1}^{N_1+t(t-2)} PA_{t-1}^n - \gamma_2 \theta_{t-1} - \gamma_3 r_{t-1} & \text{if } ST_{t-1}^{N_{t-1}+1} = 1. \end{cases} \quad (5)$$

κ_t is not known either by the Central Bank or by speculators, therefore we define PA_{t-1}^n as the predicted value of this parameter in period t by the n -th agent. The state of fundamentals is not known in the period of decision making, therefore the expected payoff is calculated:

$$E[pa_t^{N_t+1}(ST_{t-1}^{N_{t-1}+1})] = \begin{cases} 0 & \text{if } ST_{t-1}^{N_{t-1}+1} = 0, \\ v - \gamma_1 \frac{(N_1 + t(t-2))PA_{t-1}^{N_{t-1}+1}}{2} - \gamma_2 x_{t-1}^{N_{t-1}+1} - \gamma_3 r_{t-1} & \text{if } ST_{t-1}^{N_{t-1}+1} = 1. \end{cases} \quad (6)$$

Firstly we consider the border cases. We define a binary variable bad_t , which takes on value 1 if the state of fundamentals and reserves is extremely bad and even if there is “no attack” the exchange rate peg is abandoned. We also define a binary variable $good_t$, which takes on value 1 if the state of fundamentals and reserves is extremely good, so even if all speculators attack in period $t-1$, then expected payoff in period t is positive. Values of variables bad_t and $good_t$ are defined in the following way:

$$bad_t = 1 \{ (x_t^{N_t+1}, r_t) : v - \gamma_2 x_t^{N_t+1} - \gamma_3 r_t < 0 \}, \quad (7)$$

$$good_t = 1 \{ (x_t^{N_t+1}, r_t) : v - \gamma_1 (N_1 + \tau(t-2)) - \gamma_2 x_t^{N_t+1} - \gamma_3 r_t > 0 \}. \quad (8)$$

If the variable bad_t takes on value 1, then a dominant strategy for the Central Bank is to abandon the exchange rate peg even in the case of the lack of speculators attacking. Otherwise if $good_t$ equals 1, then a dominant strategy is to defend the exchange rate peg. Payoff is positive even if all speculators attack. Of course, there is no reason to attack for the speculators, if payoff from attacking (even if the attack is successful) is smaller than a transaction cost, which means that:

$$e^* - f(\theta_t) < tr. \quad (9)$$

Then the dominant strategy for speculators is to refrain from attacking.

If $bad_t + good_t = 0$, then there exists such $P\kappa_{t-1}^{N_{t-1}+1}$ that solves the following equation:

$$v = \gamma_1 (N_1 + \tau(t-2)) P\kappa_{t-1}^{N_{t-1}+1} + \gamma_2 x_{t-1}^{N_{t-1}+1} + \gamma_3 r_{t-1}, \quad (10)$$

against $P\kappa_{t-1}^{N_{t-1}+1}$. Then an optimal strategy for the Central Bank is defined as follows:

$$ST_{t-1}^{N_{t-1}+1} = 1 \left\{ P\kappa_{t-1}^{N_{t-1}+1} < \frac{2(v - \gamma_2 x_{t-1}^{N_{t-1}+1} - \gamma_3 r_{t-1})}{\gamma_1 (N_1 + \tau(t-2))} \right\}. \quad (11)$$

Similarly, as in the case of the Central Bank, the critical value $P\kappa_{t-1}^n$ is defined for each speculator. If inequality (9) is not satisfied, then an optimal strategy for speculators is given by the following formula:

$$ST_{t-1}^n = 1 \left\{ P\kappa_{t-1}^n > \frac{\frac{1}{b^n} (v - \gamma_2 x_{t-1}^n - \gamma_3 r_{t-1})}{\gamma_1 (N_1 + \tau(t-2))} \right\}. \quad (12)$$

It is further assumed that the level of reserves is known for all agents. However, neither the Central Bank nor speculators know about the initial number of speculators and their average rate of growth. Therefore, they predict these values and we define $PN_{1,t-1}^n$ and $P\tau_{t-1}^n$ as the predicted values of N_1 and τ by the n -th agent in period $t-1$. Finally, the strategies of the Central Bank and the strategies of speculators are based on their predictions. All in all, we have the following formulas concerning optimal strategy of the Central Bank and speculators:

$$ST_{t-1}^{N_{t-1}+1} = 1 \left\{ P\kappa_{t-1}^{N_{t-1}+1} < \frac{2(v - \gamma_2 x_{t-1}^{N_{t-1}+1} - \gamma_3 r_{t-1})}{\gamma_1 (PN_{1,t-1}^{N_{t-1}+1} + P\tau_{t-1}^{N_{t-1}+1} (t-2))} \right\}, \quad (13)$$

and

$$ST_{t-1}^n = 1 \left\{ P\kappa_{t-1}^n > \frac{\frac{1}{b^n} (v - \gamma_2 x_{t-1}^n - \gamma_3 r_{t-1})}{\gamma_1 (PN_{1,t-1}^n + P\tau_{t-1}^n (t-2))} \right\}, \quad n=1,2,\dots,N_{t-1}. \quad (14)$$

Genetic Algorithm in the Process of Learning

In the first period for each agent predicts the number of speculators and values of variables $PN_{1,1}^1, PN_{1,1}^2, \dots, PN_{1,1}^{N_1+1}$. Similarly predictions $P\tau_1^1, P\tau_1^2, \dots, P\tau_1^{N_1+1}$ are purely random for each agent. Distributions of these parameters are considered in section 5. In the second period, the Central Bank knows the values of N_1 and $\sum_{n=1}^{N_1} PA_1^n$, and it assumes that in the second period this value equals:

$$\sum_{n=1}^{N_2} PA_2^n = \sum_{n=1}^{N_1} PA_1^n + 0.5 P\kappa_2^{N_2+1} P\tau_2^{N_2+1}, \quad (15)$$

where $P\tau_2^{N_2+1} = P\tau_1^{N_1+1}$ and $P\kappa_2^{N_2+1} = \frac{\sum_{n=1}^{N_1} PA_1^n}{N_1}$. In the third period the Central Bank knows the values of N_1 and $\sum_{n=1}^{N_1} PA_1^n$. After the third period the Central Bank predicts a growth of the speculators' number and κ_t period according to the following formula:

$$P\tau_t^{N_t+1} = \frac{\sum_{s=2}^{t-1} (N_s - N_{s-1})}{t-1}, \quad (16)$$

And

$$P\kappa_t^{N_t+1} = \frac{\sum_{n=1}^{N_t} PA_t^n}{N_t}. \quad (17)$$

Since the foreign exchange market is not fully transparent, we assume that the speculators do not have any information concerning the number of speculators attacking in the previous period, but they observe their own payoffs and if payoff for the first speculator is higher than payoff for the second one, then this first speculator has higher chance to „survive” than the second one. It is obvious that speculators with higher payoffs are satisfied with their decisions and they do not have any incentive to change the tactic. The speculators with negative payoffs decide to change their tactic. They learn the tactic from the speculators with the positive payoffs. New speculators enter the foreign exchange market. The dynamics on the foreign exchange market imitates nature, where only correctly fitted species survive. The species that are not able to adapt to the environment are replaced by the species that are better fitted. Crossover can be interpreted as a knowledge exchange. Considering all this, foreign exchange market can be modelled as an evolving system of the autonomous interacting agents and hence the genetic algorithm can be applied here. Analysing formula for an optimal strategy of the n -th speculator in period t , we can notice that this strategy depends on: $P\kappa_{t-1}^n, v, \gamma_1, \gamma_2, \gamma_3, PN_{1,t-1}^n$ and $P\tau_{t-1}^n$. Speculators do not know any values of $\gamma_1, \gamma_2, \gamma_3$, therefore we define $Pv_t^n, P\gamma_{1,t}^n, P\gamma_{2,t}^n, P\gamma_{3,t}^n$ as predicted values of these parameters by them. We

have to choose minimum and maximum values for the predicted values of parameters associated with the cost function. These bounds are as follows:

$$\forall_{t \in \{2,3,\dots\}} \forall_n P\kappa_{t-1}^n \in \langle 0,1 \rangle, \quad (18)$$

$$\forall_{i,n} P\gamma_{i,t}^n \in \langle P\gamma_i^{\min}, P\gamma_i^{\max} \rangle, \quad i=1,2,3, \quad (19)$$

$$\forall_{t \in \{2,3,\dots\}} \forall_n PN_{1,t-1}^n \in \langle PN_1^{\min}, PN_1^{\max} \rangle \wedge \forall_{t \in \{2,3,\dots\}} \forall_n PN_{1,t-1}^n \in Z, \quad (20)$$

$$\forall_{t \in \{2,3,\dots\}} \forall_n P\tau^n \in \langle P\tau^{\min}, P\tau^{\max} \rangle \wedge \forall_{t \in \{2,3,\dots\}} \forall_n P\tau^n \in Z. \quad (21)$$

Using bounds (18)-(21) and defining the level of precision ε , we have the following formula for the number of gens in one chromosome:

$$\begin{aligned} GENS = & GENS_KAP + GENS_GAM1 + GENS_GAM2 + \\ & + GENS_GAM3 + GENS_N1 + GENS_TAU, \end{aligned} \quad (22)$$

where:

$$GENS_KAP = \left\lceil \log_2 \frac{1}{\varepsilon} \right\rceil + 1 - 1 \left\{ \log_2 \frac{1}{\varepsilon} \in Z \right\}, \quad (23)$$

$$GENS_GAM1 = \left\lceil \log_2 \frac{(P\gamma_1^{\max} - P\gamma_1^{\min})}{\varepsilon} \right\rceil + 1 - 1 \left\{ \log_2 \frac{(P\gamma_1^{\max} - P\gamma_1^{\min})}{\varepsilon} \in Z \right\}, \quad (24)$$

$$GENS_GAM2 = \left\lceil \log_2 \frac{(P\gamma_2^{\max} - P\gamma_2^{\min})}{\varepsilon} \right\rceil + 1 - 1 \left\{ \log_2 \frac{(P\gamma_2^{\max} - P\gamma_2^{\min})}{\varepsilon} \in Z \right\}, \quad (25)$$

$$GENS_GAM3 = \left\lceil \log_2 \frac{(P\gamma_3^{\max} - P\gamma_3^{\min})}{\varepsilon} \right\rceil + 1 - 1 \left\{ \log_2 \frac{(P\gamma_3^{\max} - P\gamma_3^{\min})}{\varepsilon} \in Z \right\}, \quad (26)$$

$$GENS_N1 = \left\lceil \log_2 (PN_1^{\max} - PN_1^{\min}) \right\rceil + 1 - 1 \left\{ \log_2 (PN_1^{\max} - PN_1^{\min}) \in Z \right\}, \quad (27)$$

$$GENS_TAU = \lceil \log_2(P\tau^{\max} - P\tau^{\min}) \rceil + 1 - 1\{\log_2(P\tau^{\max} - P\tau^{\min}) \in Z\}, \quad (28)$$

where $\lceil x \rceil$ denotes integer value of x . For example $GENS_KAP$ gens represent strategy concerning value of parameter κ_i etc. In order to introduce crossover and mutation, we define the quantities $pc(P\kappa)$, $pc(P\gamma_1)$, $pc(P\gamma_2)$, $pc(P\gamma_3)$, $pc(PN_1)$, $pc(P\tau)$, which denote probabilities of crossover of two chromosomes for a given parameter. $pm(P\kappa)$, $pm(P\gamma_1)$, $pm(P\gamma_2)$, $pm(P\gamma_3)$, $pm(PN_1)$, $pm(P\tau)$ denote probability of mutation of respective gens and $pr(P\kappa)$, $pr(P\gamma_1)$, $pr(P\gamma_2)$, $pr(P\gamma_3)$, $pr(PN_1)$, $pr(P\tau)$ denote proportion of gens in appropriate part of chromosome, which are crossed over.

Before we calculate fitness function, for simplicity we define the following variables:

$$SS_t^n = 1\{e^* - f(\theta_t + \varepsilon_t^n) - tr > 0\}, \quad n = 1, \dots, N_t, \quad t = 1, 2, \dots, \quad (29)$$

$$SB_t = 1\left\{v - \gamma_1 \sum_{n=1}^{N_t + \tau(t-1)} PA_t^n - \gamma_2(\theta_t + \varepsilon_t^{N_t+1}) - \gamma_3 r_t > 0\right\}, \quad t = 1, 2, \dots, \quad (30)$$

$$SBS_t^n = 1\left\{v - \gamma_1 \sum_{n=1}^{N_t + \tau(t-1)} PA_t^n - \gamma_2(\theta_t + \varepsilon_t^n) - \gamma_3 r_t > 0\right\}, \quad t = 1, 2, \dots, \quad n = 1, \dots, N_t. \quad (31)$$

It can be easily seen that the n -th speculator considers attacking in period t only if $SS_t^n = 1$. Otherwise transaction costs exceed payoff and the attack is not taken into account. The value of variable SB_t informs about the strategy of the Central Bank. It abandons the exchange rate peg if this variable takes on the value 0, and defends it otherwise. Variable SBS_t^n can be interpreted as expected (by the n -th speculator) strategy of the Central Bank. This variable takes on value 1 if the n -th speculator predicts that the Central Bank will defend the exchange rate peg and 0 otherwise. Since payoff may be negative, we do monotonic transformation in order to calculate the value of the fitness function:

$$F_t^n = \exp(E(\text{pay}_{t+1}^n)). \quad (32)$$

The speculators attacks with the probability b^n if $SS_t^n = 1$, $SB_t^n = 0$ and $SBS_t^n = 0$. Therefore, the fitness function in our model is given in the following formula:

$$F_t^n = \begin{cases} b^n \exp(-tr) + (1 - b^n) & \text{if } SS_t^n SB_t^n = 1 \wedge SBS_t^n = 0, \\ \exp(0) = 1 & \text{if } SS_t^n = 0, \\ b^n \exp(e * -f(\theta_t) - tr) & \text{if } SS_t^n = 1 \wedge (1 - SB_t^n)(1 - SBS_t^n) = 1. \end{cases} \quad (33)$$

We use roulette-wheel selection method in order to choose appropriate chromosomes in the next period.

Results of the Simulation

Experiment

In the first simulation experiment, the probability of abandoning the exchange rate peg is calculated. If after 20 periods the Central Bank does not change its strategy and still defends the exchange rate peg or dominant strategy for speculators is not to attack, then we assume that the exchange rate peg is abandoned. The experiment is done for different states of fundamentals (weak fundamentals, medium fundamentals, strong fundamentals), different levels of reserves (low reserves, medium reserves, high reserves) and different variants of the speed of appearance of new speculators.

Initial number of speculators equals $N_1 = 10$ and predicted numbers of speculators in the first period are calculated according to the following formulas:

$$P(PN_1^n = k) = \begin{cases} \frac{1}{6} & \text{if } k = 7, 8, 9, 10, 11, 12, 13, \\ 0 & \text{otherwise} \end{cases}, \quad (34)$$

for $n = 1, 2, \dots, N_1$ and

$$P(PN_1^{N_1+1} = k) = \begin{cases} \frac{1}{3} & \text{if } k = 9, 10, 11, \\ 0 & \text{otherwise} \end{cases}, \quad (35)$$

b^n and $P\kappa_1^n$ are selected randomly from $U(0,1)$ for all agents. Similarly, $P\tau_1^n$ is selected randomly for all agents in the first period. We assume that for $n = 1, \dots, N_1$, when $\tau = 1$:

$$P(P\tau_1^n = k) = \begin{cases} \frac{1}{3} & \text{if } k = 0, 1, 2, \\ 0 & \text{otherwise} \end{cases}, \quad (36)$$

and when $\tau = 4$

$$P(P\tau_1^n = k) = \begin{cases} \frac{1}{3} & \text{if } k = 3, 4, 5, \\ 0 & \text{otherwise} \end{cases}, \quad (37)$$

Table 2 presents all variants considered in our experiment. We consider weak, medium and strong fundamentals, low, medium and high level of reserves and low and high speed of appearance of new speculators.

Table 2. Variants of the simulation experiment

Variant	Speed of appearance of new speculators	Reserves	Fundamentals
1	$\tau = 1$	$r_t = 1 + 0.01t$	$\theta_t = 0.5 + 0.01t$
2	$\tau = 1$	$r_t = 1 + 0.01t$	$\theta_t = 1 + 0.01t$
3	$\tau = 1$	$r_t = 1 + 0.01t$	$\theta_t = 1.8 + 0.01t$

Table 2 continued

Variant	Speed of appearance of new speculators	Reserves	Fundamentals
4	$\tau = 1$	$r_t = 3 + 0.01t$	$\theta_t = 0.5 + 0.01t$
5	$\tau = 1$	$r_t = 3 + 0.01t$	$\theta_t = 1 + 0.01t$
6	$\tau = 1$	$r_t = 3 + 0.01t$	$\theta_t = 2 + 0.01t$
7	$\tau = 1$	$r_t = 5 + 0.01t$	$\theta_t = 0.5 + 0.01t$
8	$\tau = 1$	$r_t = 5 + 0.01t$	$\theta_t = 1 + 0.01t$
9	$\tau = 1$	$r_t = 5 + 0.01t$	$\theta_t = 2 + 0.01t$
10	$\tau = 4$	$r_t = 1 + 0.01t$	$\theta_t = 0.5 + 0.01t$
11	$\tau = 4$	$r_t = 1 + 0.01t$	$\theta_t = 1 + 0.01t$
12	$\tau = 4$	$r_t = 1 + 0.01t$	$\theta_t = 1.8 + 0.01t$
13	$\tau = 4$	$r_t = 3 + 0.01t$	$\theta_t = 0.5 + 0.01t$
14	$\tau = 4$	$r_t = 3 + 0.01t$	$\theta_t = 1 + 0.01t$
15	$\tau = 4$	$r_t = 3 + 0.01t$	$\theta_t = 1.8 + 0.01t$
16	$\tau = 4$	$r_t = 5 + 0.01t$	$\theta_t = 0.5 + 0.01t$
17	$\tau = 4$	$r_t = 5 + 0.01t$	$\theta_t = 1 + 0.01t$
18	$\tau = 4$	$r_t = 5 + 0.01t$	$\theta_t = 1.8 + 0.01t$

Source: own calculations.

Table 3 presents the assumptions concerning values of remaining parameters:

Table 3. Assumptions concerning remaining parameters

Parameter	e^*	v	γ_1	γ_2	γ_3	$P\gamma_1^{\min}$	$P\gamma_1^{\max}$	$P\gamma_2^{\min}$	$P\gamma_2^{\max}$
Value	2	2	0.8	-0.7	-0.7	0	2	-2	0
Parameter	$P\gamma_1^{\min}$	$P\gamma_1^{\max}$	β_t	$\tilde{\beta}_t$	$f(\theta_t)$	ε			
Value	-2	0	$5t$	$10t$	θ_t	0.01			

Source: own calculations.

We used software R 3.1.2 in order to conduct this simulation experiment. We made 10000 replications and calculated probability of defending the exchange rate peg and mean payoff for speculators for different variants. In table 4 there are calculated probabilities of defending the exchange rate peg presented, while Table 5 contains mean payoff.

Table 4. Probability of defending the exchange rate peg for different variants

		$\tau = 1$			$\tau = 4$		
Fundamentals		Weak	Medium	Strong	Weak	Medium	Strong
Reserves							
Low		0.323	0.346	0.370	0.134	0.157	0.189
Medium		0.432	0.433	0.459	0.173	0.181	0.197
High		0.470	0.472	0.535	0.225	0.236	0.291

Source: own calculations.

Table 5. Mean payoff for speculators for different variants

		$\tau = 1$			$\tau = 4$		
Fundamentals		Weak	Medium	Strong	Weak	Medium	Strong
Reserves							
Low		0.13	0.03	-0.04	0.43	0.13	-0.09
Medium		0.04	-0.11	-0.19	0.15	-0.23	-0.19
High		-0.05	-0.17	-0.29	-0.21	-0.25	-0.31

Source: own calculations.

In the case of low level of reserves and weak macroeconomic fundamentals, the probability that the exchange rate peg collapses is about 0.68 or 0.87 depending on the speed of appearance of new speculators. When the Central Bank increases the level of reserves and macroeconomic fundamentals improve, then the probability of the fixed exchange rate regime collapse decreases. However, the probability of defending the exchange rate peg strongly depends on the speed of appearance of new speculators. This result shows that in the case of panic and herding behaviour of speculators, good macroeconomic situation may not help. Keeping a high level of

reserves and concentrating on the state of macroeconomic fundamentals seems to be important, however it does not guarantee efficient defending of the exchange rate peg. Analysis of the payoffs of speculators shows that faster appearance of them and better cooperation gives advantage in the case of weak fundamentals and low level of reserves. In the case of better state of reserves and macroeconomic fundamentals, bad decisions of speculators dominate, and most of them pay transaction costs and do not receive payoffs.

Conclusions

The model seems to catch reality in a more complex way: the level of noise changes in time (decreasing), there are different states of fundamentals (with “more sensitive” upper part of the scale), number of inflowing agents can be low or high (due to different globalization phases, different capital flow phases, different uncertainty levels).

Dynamic nature of the model is also reflected in defining some kind of continuity of CB and agent behavior. Both sides must formulate their strategies in a continuous way, and therefore simultaneously, which is a shift from a single-action approach (single nonreplicable attack, located in the short run and therefore sequential decision making process) to a longer perspective.

In fact, the results are in line with intuition, which may confirm that the usage of genetic algorithm was a right decision. A weaker level of fundamentals decreases probability of defending the exchange rate peg and increases pay-off for speculators. If dynamics of inflow of agents is higher, then the probability of defending peg is lower, the pay-off for speculators is higher and the mean time of peg maintenance duration is lower. The higher the level of international reserves the higher probability of peg defending, the lower pay-off for speculators and the higher duration of peg maintenance.

Satisfying results of our simulation based on the usage of genetic algorithm encourage furthering works on constructing the model that could give more specific parameters of the speculative attack occurrence probability. Following this direction may challenge other warning signal models already existing in the literature.

References

- Angeletos, G. M., Hellwig, C., & Pavan, A. (2006). Signaling in a Global Game: Coordination and Policy Traps. *Journal of Political Economy*, 114(3).
- Arifovic, J., & Maschek, M. K. (2012). Currency Crisis: Evolution of Beliefs and Policy Experiments. *Journal of Economic Behavior & Organization*, 82(1). DOI:10.1016/j.jebo.2012.01.001.
- Aydin, A. D., & Cavdar, S. C. (2015). Prediction of Financial Crisis with Artificial Neural Network: An Empirical Analysis on Turkey. *International Journal of Financial Research*, 6(4). DOI: 10.5430/ijfr.v6n4p36.
- De Grauwe, P., & Grimaldi, M. (2006). *The Exchange Rate in a Behavioral Finance Framework*. Princeton University Press.
- Fama, E. F. (1970). Efficient Capital Markets: A Review of Theory and Empirical Work. *Journal of Finance*, 25(2).
- Frydman, R., & Goldberg, M. D. (2007). *Imperfect Knowledge Economics: Exchange Rates and Risk*. Princeton University Press.
- Lyons, R. K. (2001). *The Microstructure Approach to Exchange Rates*. Cambridge: MA: MIT press.
- Morris, S., & Shin, H. S. (1998). Unique Equilibrium in a Model of Selffulfilling Currency Attacks. *American Economic Review*, 88(3).
- Sarlin, P. (2014). On Biologically Inspired Predictions of the Global Financial Crisis. *Neural Computing and Applications*, 24(3-4).
- Sarno, L., & Taylor, M. P. (2002). *The Economics of Exchange Rates*. Cambridge: Cambridge University Press.
- Tversky, A., & Kahneman, D. (1991). Loss Aversion in Riskless Choice: A Reference-dependent Model. *Quarterly Journal of Economics*, 106(4).