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**Impact of a Modified HP Filter on Countercyclical Behavior of the Swiss Fiscal Rule***

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**Keywords:** fiscal rule; countercyclical policy

**Abstract:** Fiscal rules are an idea that has been getting more attention lately due to the recent economic crisis. Fiscal rules have been tested for many properties, including countercyclical behavior. The present paper focuses on the Swiss fiscal rule and investigates the impact of a modified HP filter, used in the rule, on countercyclical behavior of the rule. The paper uses real GDP time series for over a hundred countries and applies the rule to each time series in two variants, with a standard HP filter and a modified HP filter. For each result procyclicality indices are calculated. It is found that the modification of HP filter, used in the Swiss fiscal rule, increases countercyclical behavior of the rule.

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Introduction

Fiscal rules are an idea that has been getting more and more popular nowadays as the recent economic crisis, together with aging of societies, is posing serious challenges for public finance. Their perception as tools effectively curbing public deficits leads to their implementation in numerous countries, cf. Schaechter et al. (2012). Any fiscal rule can be judged with respect to some basic criteria, cf. Wyplosz (2012), Kopits (2001), including particularly its capability to stabilize debt and pursue anticyclical fiscal policy, flexibility which allows for reactions to exogenous shocks or simplicity which allows members of the public to understand it. There is some empirical evidence confirming the efficiency of fiscal rules with respect to the first two aforementioned issues, cf. Kumar et al. (2009), Holm-Hadulla et al. (2012).

This paper focuses on the Swiss fiscal rule. Behavior of the rule with respect to debt stabilization and its countercyclical has been tested previously using artificial data, cf. Geier (2012). The current research focuses on the impact of a modified HP filter, proposed by Bruchez (2003) and used in the Swiss fiscal rule, on the countercyclical behavior of the Swiss rule using empirical real GDP time series from over a hundred countries. The method used, described in details later, is straightforward. It applies two rules, one with the standard HP filter and the other with a modified HP filter, to each real GDP time series and then computes two measures of procyclicality for each rule. Finally, these measures are compared to see the effects of the modification. In particular, the current paper is not focused on testing if the Swiss fiscal rule itself leads to a countercyclical fiscal policy, even though as a side result we confirm that for a majority of all cases the rule, in fact, leads to a countercyclical fiscal policy.

The structure of the paper is the following. In the section Method the method used in the paper is described in details. The section Swiss Fiscal Rule introduces the Swiss fiscal rule together with implementation details. Finally, section Results and Discussion provides results of computations and a discussion of results. Section Conclusions concludes.

Method

All results provided in the current paper were computed\(^1\) using the latest version of Mathematica, cf. Wolfram Research (2015). The calculations

\(^1\) The full Mathematica code is available on request from M. Ramsza.
required three steps. The first step was to write a module calculating government expenditure limits according to mathematical specification of the Swiss fiscal rule. This step was further split into separate tasks including programming the standard HP filter and the modified HP filter, programming business cycle adjustment factor $k$ and so on. Particular steps are clear from the specification of the fiscal rule provided in section Swiss Fiscal Rule.

The next step involved collecting data. All computations are based on real GDP time series (expressed in 2000 USD). The data was retrieved from the curated database of economic data provided by Wolfram Research, Inc. Initially, the data for all countries were retrieved (247 countries). Subsequently, only time series containing at least 48 years of data were used in computations resulting in 101 countries. The span of 48 years was selected due to the 24 year rolling window used in the Swiss fiscal rule (described in details below).

The last step comprised all calculations. Two different rules were used: one with the standard HP filter and one with the modified HP filter (rules and filters are defined in the section Swiss Fiscal Rule). Each rule was applied to all real GDP time series. These resulted in two time series of budget expenditure's limits for each real GDP time series. For each such series two measures of procyclicality were computed. This procedure produced four samples, each of size 101, of procyclicality measures: two samples related to the standard HP filter and two related to the modified HP filter. The resulting samples were compared to see how the modification of the HP filter changes the procyclicality of the fiscal policy defined by the rule.

As pointed in Fatás & Mihov (2010) there is no consensus on how fiscal cyclicality should be measured. The simplest measure is a correlation between a cyclical component of output, here measured as a difference between the real GDP and the trend derived through application of an HP filter, and a fiscal variable, which here is budget expenditures' limits, as used in e.g. Kaminsky et al. (2005) or Talvi & Vegh (2005). However, as argued in Lane (2003), Woo (2009), regression based indicators tend to be more precise. That is why a second measure is introduced, following Alesina et al. (2008), based on the following regression

\[
\Delta \log G_t = \alpha_0 + \alpha_1 \log \text{output gap}_t + \alpha_2 \log G_{t-1} + \alpha_3 t + \epsilon_t, \tag{1}
\]

where $\Delta$ is the operator of first difference and $G_t$ is a fiscal variable. Output gap is calculated as a difference between the real GDP and its trend derived from an application of the HP filter. Parameter $\alpha_1$ is taken as the procycli-
cality indicator with the interpretation that a larger \( \alpha_1 \) means a larger procyclicality.

There are also other regression-based indicators, e.g. Lane (2003), Thornton (2008), Woo (2009), with very similar constructions, usually substituting output gap with a first difference of real GDP or introducing higher powers of time trend into the equation (1), e.g. Halland & Bleaney (2009). We select the two measures described above because they are consistent in the first one calculating correlation between output gap and a fiscal variable, while the second one corrects this indicator with two other covariates.

**Swiss Fiscal Rule**

The Swiss fiscal rule is essentially a single equation relating previous GDP path and future maximum budget expenses. The equation reads

\[
G_t = k_t R_t, \quad (2)
\]

where \( G_t \) is an upper limit to budget expenditures\(^2\), \( R_t \) denotes budget revenues and \( k_t \) is a business cycle adjustment factor. The main idea behind equation (2) is obvious. If for all \( t \) values of adjustment factor \( k_t = 1 \), then expenditures are equal to revenues and the budget deficit is always zero. This, however, leads to a procyclical fiscal policy. The cycle adjustment factor \( k_t \) is used to counter this behavior within a business cycle by allowing deficits or necessitating surpluses in contraction and expansion phases, respectively.

The adjustment factor is calculated using an HP filter. The HP filter defines a trend \( y^*_t \) in a time series \( y_t \) such as to minimize the following functional

\[
C = \frac{1}{\lambda} \sum_{t=-N+T+1}^{T} (y_t - y^*_t)^2 + \sum_{t=-N+T+2}^{T-1} \left( (y^*_{t+1} - y^*_t) - (y^*_t - y^*_{t-1}) \right)^2,
\]

\(^2\) It is assumed that the expenditures always reach their allowed upper limit.
where $N$ is the length of a rolling window. The first part is an “error” that is made while substituting the original time series for a trend time series. The second part reflects “smoothness” of the trend time series. The $\frac{1}{\lambda}$ coefficient balances two parts, for small $\lambda$ the trend part is very close to the original time series $y_t$, while for large $\lambda$ the trend part is close to being linear.

The Swiss fiscal rule uses a different version of the HP filter described in Bruchez (2003). This version differs only at the boundaries of a rolling window to which the filter is applied. This version defines the trend part $y_t^*$ of the time series $y_t$ in order to minimize the following functional

$$
C = \sum_{t=-N+T+1}^{T} \frac{1}{\lambda_t} (y_t - y_t^*)^2 + 
$$

$$+
\sum_{t=-N+T+2}^{T-1} ((y_{t+1}^* - y_t^*) - (y_{t}^* - y_{t-1}^*))^2,
$$

where

$$
\lambda_t = \begin{cases} 
3 \lambda & \text{for } t = T - N + 1 \text{ and } t = T \\
\frac{3}{2} \lambda & \text{for } t = T - N + 2 \text{ and } t = T - 1 \\
\lambda & \text{for other } t
\end{cases}
$$

The above modification leads to larger values of $\lambda$ at the boundaries of a rolling window, which further leads to a trend part being more linear at the boundaries of a rolling window.

The time series of the real GDP (in 2000 US dollars) are denoted by $Y_t$. Logs of real GDP are denoted by $y_t = \log Y_t$. The adjustment factor $k_t$ is defined as

$$
k_t = \frac{\exp y_t^*}{y_t},
$$

where $y_t^*$ is a value at time $t$ of a trend calculated using an HP filter. Specifically, the trend is calculated by applying an HP filter, or its modified
version, to a 24 year rolling window\(^3\) starting at time \(t - N + 1\) and ending at time \(t\), which is \(N = 24\) in equations (3) and (4), and \(y^*_t\) is the value of the trend at time \(t\).

The idea behind this definition of a business cycle adjustment factor \(k_t\) is that if a business cycle is at a peak, then the adjustment factor is less than 1, and if a business cycle is at a trough, then the adjustment factor is more than 1. Effectively, during expansion expenditures are smaller than revenues, but during contraction expenditures are larger than revenues. The use of the modified version of the HP filter is meant to increase this effect. The above construction may not give a countercyclical fiscal policy depending on a time series, but also on a particular measure of procyclicality. This paper abstracts from those considerations, because the main focus is the impact on the procyclicality of the HP filter's modification in comparison with the standard HP method.

The last thing that needs to be defined in order to test the fiscal rule (2) is an effective way of calculating revenues \(R_t\). Considering the fact that, for most countries, revenues constitute on average just a constant share of GDP the next definition, following Geiger (2012), is used in all calculations herein

\[
R_t = \beta Y_t. \tag{6}
\]

The above formulas require at least two constants to be defined for any calculations. First one is the smoothing factor \(\lambda\). Following Swiss government, the value \(\lambda = 100\) was used, which is a typical choice for annual data. The second one is \(\beta\). There is no good choice here, since this parameter varies across countries. The value of \(\beta = 0.3\) was used in all calculations.

**Results and Discussion**

Figure 1 shows behavior of all real GDP time series, while Figure 2 shows behavior of all time series of logs of real GDP. Behavior of most time series is typical with logs resembling linear trends.

\(^3\) The length of the window equal to 24 seems to be just a discrentional choice of Swiss fiscal authorities.
Figure 1. Behavior of real GDP for all countries in USD, base year is 2000

Source: own calculation.

Figure 2. Behavior of logs of real GDP for all countries

Source: own calculation.
Figure 3 and 4 show behavior of business cycle adjustment factors $k_t$. Figure 3 shows behavior of adjustment factors resulting from the use of the standard HP filter, while figure 4 shows behavior of adjustment factors resulting from the use of the modified HP filter.

**Figure 3.** Behavior of adjustment factors calculated using a standard HP filter

Source: own calculation.

**Figure 4.** Behavior of adjustment factors calculated using a modified HP filter

Source: own calculation.
Adjustment factors are calculated using formula (5). Formula (5) requires the use of a trend component of real GDP time series. Those trend components are calculated as described above once with a standard HP filter, defined by (3), and once with a modified HP filter, defined by (4). In both cases, the parameter $\lambda$ is set to 100, as mentioned earlier.

Quick comparison of Figures 2(a) and 2(b) shows that behavior of both types of adjustment factors is qualitatively similar. However, comparison of scales shows that the modified adjustment factors seem to be a larger spread of the standard adjustment factors. In order to better compare the behavior of both types of adjustment factors, those types are further compared for the case of the USA.

Figure 5 and 6 show trend components for the US data and both types of adjustment factors. Figure 5 shows the original real GDP for the USA (blue line) together with a standard trend component (yellow line) and a modified trend component (green line). Related to that figure is Figure 6, showing a standard adjustment factor (blue line), and a modified adjustment factor (yellow line). The behavior of both types of adjustment factors is qualitatively similar. If a trend component is above the time series (contraction) then an adjustment factor is above 1, if a trend is below the time series (expansion) then an adjustment factor is below 1. The difference between the types of adjustment factors stems from different filters used to calculated trend components. The modified adjustment factor is more extreme.

**Figure 5.** Trends calculated using the standard HP filter and the modified version of HP filter

![Trends comparison](image)
Figure 6. Behavior of an adjustment factor calculated using the standard HP filter and the modified version of an HP filter. Time series for USA

Source: own calculation.

In order to check how the use of the modified HP filter affects counter-cyclical behavior of the fiscal rule, the following calculations were made, as outlined previously. For each real GDP time series a rule with a standard HP filter and a rule with a modified HP filter were applied resulting in two budget expenditures' limits time series. For each of those, two measures, as defined above, were calculated resulting in four sample of size 101. Table 1 shows basic statistics for both samples.

Table 1. Basic statistics for procyclicality measures

<table>
<thead>
<tr>
<th></th>
<th>measure 1</th>
<th>measure 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>standard</td>
<td>modified</td>
</tr>
<tr>
<td>mean</td>
<td>-0.129</td>
<td>-0.165</td>
</tr>
<tr>
<td>standard deviation</td>
<td>0.338</td>
<td>0.345</td>
</tr>
<tr>
<td>1st quartile</td>
<td>-0.424</td>
<td>-0.451</td>
</tr>
<tr>
<td>median</td>
<td>-0.155</td>
<td>-0.208</td>
</tr>
<tr>
<td>3rd quartile</td>
<td>0.156</td>
<td>0.121</td>
</tr>
<tr>
<td>max</td>
<td>0.580</td>
<td>0.578</td>
</tr>
<tr>
<td>min</td>
<td>-0.773</td>
<td>-0.800</td>
</tr>
</tbody>
</table>

Source: own calculations.
Inspection of Table 1 shows that for both measures all basic statistics related to the samples calculated with the modified rule are smaller than the respective statistics for the samples calculated with a rule using the standard version of an HP filter. This effect shows that the modified rule improves upon countercyclical behavior of fiscal policy.

To further investigate the impact of the modified HP filter, both pairs of samples, one pair for each procyclicality measure, were plotted. Figure 7 and 8 show plots for both measures. On each plot, an identity line is plotted together with procyclicality measures for the modified rule against the standard rule.

For the first measure, almost all points are below the identity line. The only exceptions are Bangladesh, Brazil, Ghana, Panama, Sri Lanka and Zambia. All these are developing countries, and even for those countries the negative impact is very small, as the maximum increase is around 0.02 (for Ghana). For the second measure the impact is more visible and all points are shifted below the identity line. Also, it seems that for countries with higher values of procyclicality measure the impact is more dramatic.

**Figure 7.** Impact of the modification of an HP filter on procyclicality measures of countercyclical behavior of budget expenditures' limits. Measure 1

Source: own calculation.
Figure 8. Impact of the modification of an HP filter on procyclicality measures of countercyclical behavior of budget expenditures' limits. Measure 2

Two linear regressions were run (dashed lines on figure 4) and in both cases they were highly statistically significant. For data resulting from the use of a first procyclicality measure the following result was achieved

\[ \text{modified} = -0.034 + 1.017 \text{ standard}, \]

where \(p\)-values were all below \(10^{-17}\). That is, with respect to the first measure an average impact is about \(-0.034\). For data resulting from the use of a second procyclicality measure the following result was achieved

\[ \text{modified} = -0.240 + 0.757 \text{ standard}, \]

where \(p\)-values were below \(10^{-51}\). That is, with respect to the second measure an average impact is about \(-0.240\).
Conclusions

The calculations reported above show that the Swiss fiscal rule, with both standard and modified HP rules, exhibits desired countercyclical behavior. This can be expected from the very construction of the rule.

Introduction of the modified version of an HP filter leads to an increased countercyclical behavior of budget expenditures' limits. This result is robust to the two measures of procyclicality used. Nevertheless, it should be pointed out that the above results may change as more countries are included into the sample used or more data is available. Also, there might be some changes as the parameter $\lambda$ takes on different values.

References


